Advanced **C**omputational and **C**ivil **E**ngineering **S**tructural **S**tudies

Heat Transfer, Conduction

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1. Introduction to Heat Transfer
2. Heat Conduction
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6. Finite Element Approach

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What is heat?

Heat is a form of energy in transit due to a temperature difference.

What is heat transfer?

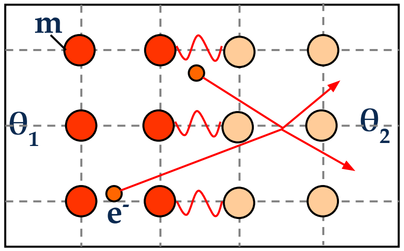
Heat transfer is energy that flows from higher to lower level of temperature without any work being performed.

In which way is the amout of transferred heat described?

flow = transport coefficient x potential gradient

- ***flow***: *heat flux q [W/m²] or heat transfer rate Q [W]*
- ***coefficient:*** *depends on transfer characteristics*
- ***gradient***: *difference resp. derivative*

Conduction

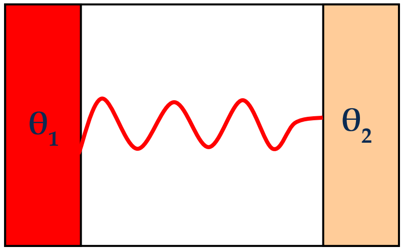


- Heat transfer in resting fluids and solids
- Diffusive transport of thermal energy
- Fluids: via moving atoms & molecules
- Solids: lattice oscillations and movement of unbound electrons (in electroconductive materials)
- Description via Fourier's law:

$$q = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

Radiation

- Energy transfer between all matters, regardless of the form of substance (liquid, gas, solid)
- Description due to wave theory (Maxwell) resp. photon emission (Plank)
- Even through vacuum possible, in contrast to conduction and convection which require presence of material medium
- Radiant heat exchange between two surfaces:

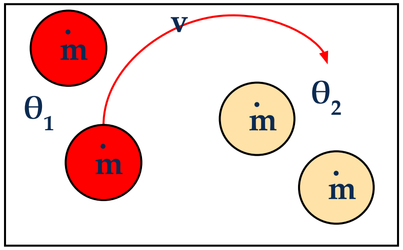


$$Q = C \cdot A \cdot (T^4{}_{\mathrm{Surface\_1}} - T^4{}_{\mathrm{Surface\_2}})$$

Convection

- Heat transfer in/by moving fluid particles
- 1st transfer via macroscopic resp. bulk motion of the fluid (advection)
- 2nd transfer due to random molecular motion (diffusion)
- The faster the fluid motion, the faster the convective heat transfer
- Two types: forced and natural convection
- Description for exchange between fluid and adjoining surface:

$$Q = \alpha \cdot A \cdot (T_{\text{Surface}} - T_{\text{Fluid}})$$



Microscopic view:

- Molecules and atoms are in mutual interaction
- Particles exchange kinetic energy in chaotic way
- Fast moving molecules collide with slower moving molecules: low-energy molecules/atoms absorb energy (temperature level increases) and high-energy molecules/atoms release energy (temperature level decreases)

Macroscopic view:

- More kinetic energy is transferred from higher to lower temperature level than vice versa
- System tends towards thermal equilibrium (homogeneous temperature level)

Heat Conduction- Simplified Approach

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Heat transfer through a solid building construction can be simplified as:

Steady- state which assumes

- time- constant boundary conditions (indoor conditions, climate conditions, heat sources or sinks,…)
- and thus no relevance of thermal storage effects

One- dimensional which assumes

- heat flux perpendicular to the construction surface area
- and thus no relevance of thermal bridge effects

Solely **heat conduction related** which assumes

- Convection or radiation transfer can be neglected or described via thermal conductivity (e.g. air layers)

Heat Conduction: 1D

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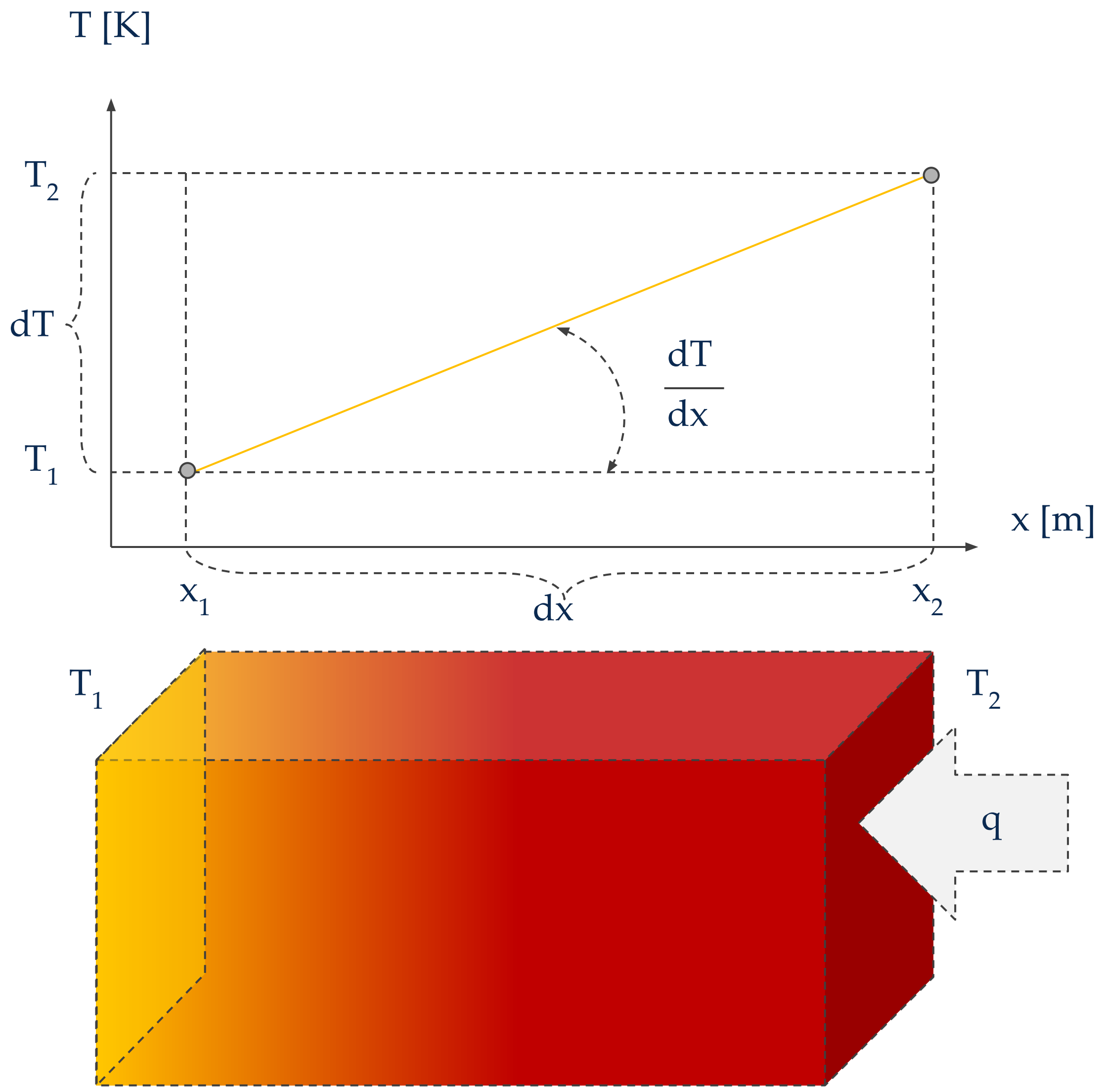


Temperature gradient

$$\frac{T_2 - T_1}{x_2 - x_1} = \frac{dT}{dx} \left[\frac{W}{K}\right]$$

Resulting heat flux

$$q = \frac{dT}{dx} \cdot -\lambda \left[\frac{W}{m^2}\right]$$

Thermal conductivity

$$\lambda \left[\frac{W}{mK}\right]$$

Resulting 1-d steady state heat flux is:

$$q = \frac{dT}{dx} \cdot -\lambda \quad \left[\frac{W}{m^2}\right]$$

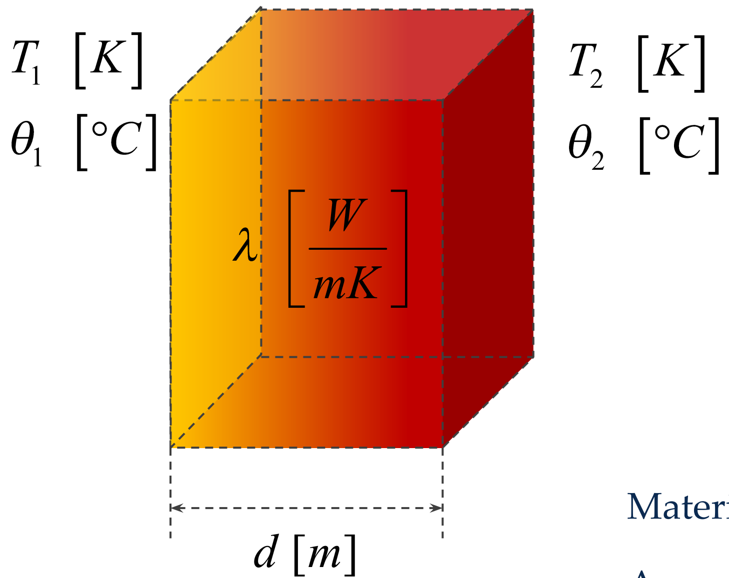
Total resulting 1-D steady- state heat transfer rate is:

$$Q = \frac{dT}{dx} \cdot -\lambda \cdot A \quad [W]$$

Resulting heat transfer rate (Q) due to heat conduction is:

- Proportional to the length- related temperature difference (temperature gradient)
- Proportional to the surface area
- Depending on solely one material property, the thermal conductivity

Heat Conduction – Thermal Conductivity



Heat transfer resistance: $R = \frac{d}{\lambda} \left[\frac{m^2K}{W}\right]$

Influence material thickness: d $\nearrow$ R $\nearrow$

Influence thermal conductivity: λ $\nearrow$ R $\searrow$

Material	l in W/mK	Material	l in W/mK
Argon (stat.)	0.016	Cellular Concrete	0.130
Air (stat.)	0.030	Brick	0.750
PS-foam	0.035	Lime sand stone	1.350
Mineral wool	0.040	Concrete	1.450
Calciumsilicate insulation	0.055	Sand stone	2.050
Wood	0.110	Steel	40.00

What is Thermal Conductivity?

Thermal conductivity is the ability of a material to conduct heat through it.

It defines the heat transfer rate [W] per distance unit [m] between two plane surfaces and per unit temperature difference [K] between these two surfaces.

What influences the thermal conductivity of a material?

1. Material density
2. Electric conduction
3. Porosity (see density)
4. Temperature
5. Pressure (Fluids)
6. Heat flow direction (anisotropic materials)

Heat Conduction: Thermal Conductivity

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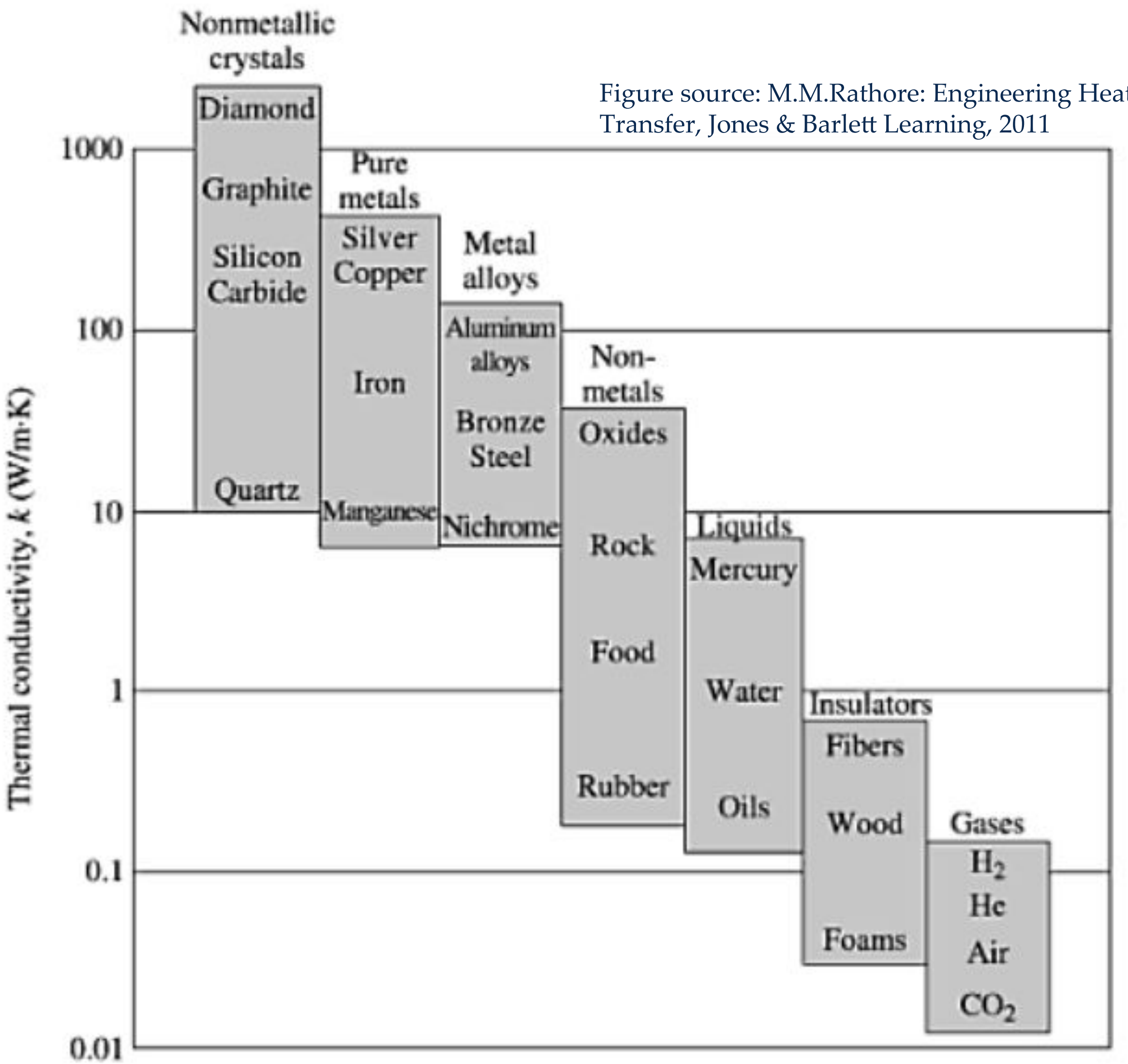

Figure source: M.M.Rathore: Engineering Heat Transfer, Jones & Barlett Learning, 2011

Thermal Conductivity and Porosity

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Porosity resp. total pore fraction is the sum of closed (non-accessible for water) and open pore (accessible for water) fraction:

- Bulk/total pore fraction $\Psi_{total} = 1 - \frac{\rho_{total}}{\rho_{solid}}$
- Open porosity $\Psi_{open} = \Psi_{total} - \Psi_{closed}$

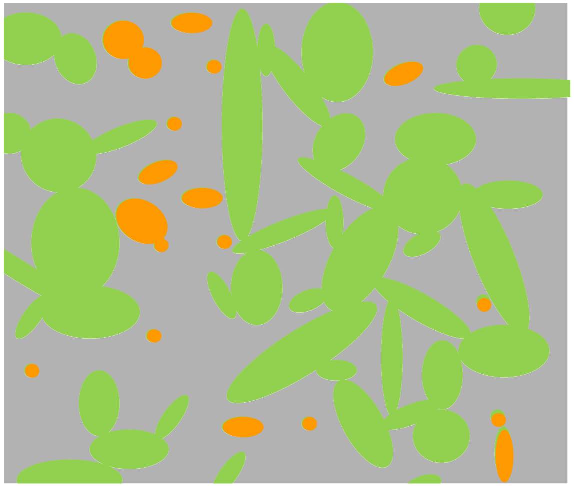
$$= \Psi_{macro} + \Psi_{micro}$$

solid volume

total pore volume

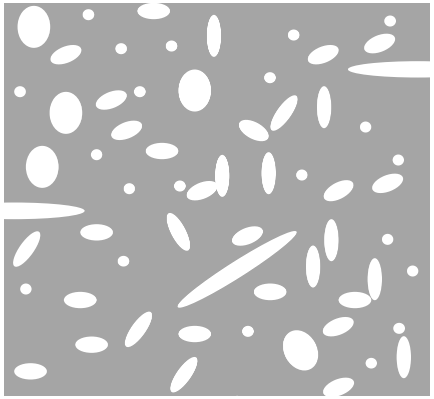
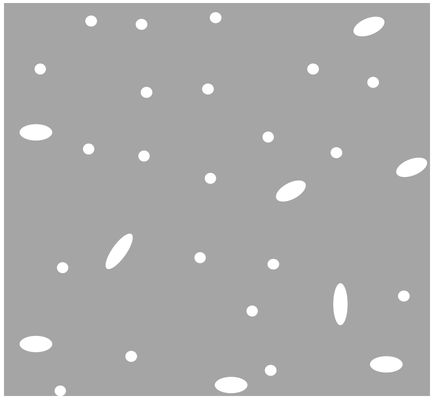
closed pore volume

open pore volume



➢ Porosity and total/gross density are inversely proportional to each other

➢ Thermal conductivity is inversely proportional to total porosity.

	High-porous brick (HPB)		Low-porous brick (LPB)	
				
Total volume V	V_{HPB}	=	V_{LPB}	$[m^3]$
Total mass m	m_{HPB}	<	m_{LPB}	$[kg]$
Solid volume V_{Solid}	$V_{Solid,HPB}$	<	$V_{Solid,LPB}$	$[m^3]$
Total pore volume $V_{Pores} = V - V_{Solid}$	$V'_{Pores,HPB}$	>	$V_{Pores,LPB}$	$[m^3]$
Solid density $\rho_{Solid} = \frac{m}{V_{Solid}}$	$\rho_{Solid,HPB}$	=	$\rho_{Solid,LPB}$	$\left[\frac{kg}{m^3}\right]$
Gross density $\rho_{total} = \frac{m}{V}$	$\rho_{Total,HPB}$	<	$\rho_{Total,LPB}$	$\left[\frac{kg}{m^3}\right]$

Thermal Conductivity and Porosity

Total volume V

Solid volume V_{Solid}

Water volume V_{Water}

Air volume V_{Air}

Open pores of a porous material (micro- to macro- pores) can be filled with water instead of air.

- This increases the entire thermal conductivity of the material
- The resulting value can be calculated as volume-weighted average:

$$\lambda = (1 - \theta_{Water}) \cdot \lambda_{dry} + \theta_{Water} \cdot \lambda_{Water}$$

with: $\theta_{Water} = \frac{V_{Water}}{V} \left[\frac{m^3}{m^3}\right]$

Conductivity and Moisture Content

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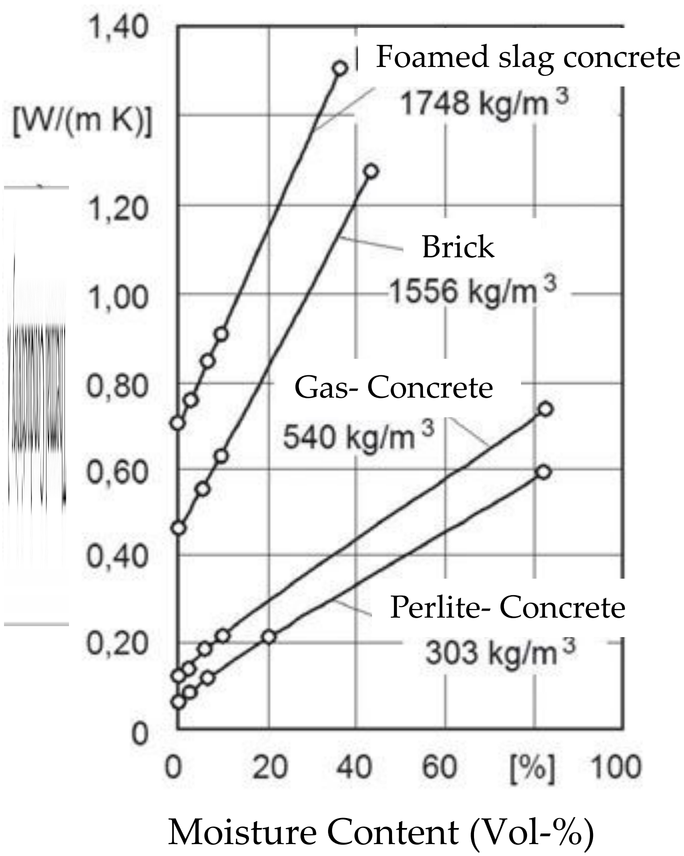

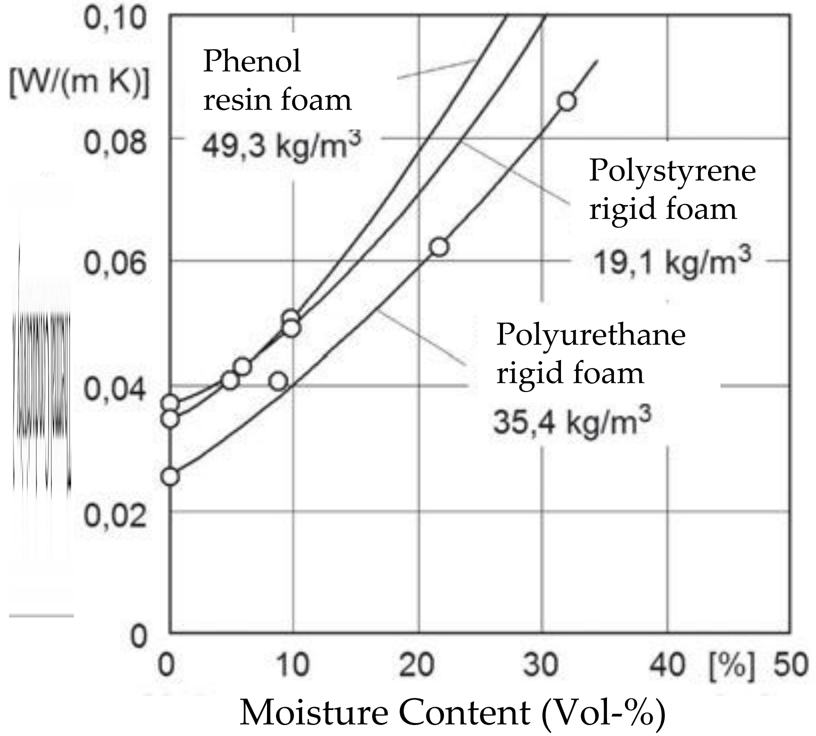

Figures source: W. M. Willems: Lehrbuch der Bauphysik – Schall, Wärme, Feuchte, Licht, Brand, Klima, Springer Vieweg Verlag, Wiesbaden, 2013

Thermal Conductivity and Temperature

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Common building materials

- Increasing thermal conductivity with temperature
- Impact small, therefore mostly neglected
- Reference values (rated values) for thermal conductivity usually given for 10°C
- High-temperature values (e.g. insulation heating systems) for 40 °C
- Values listed in DIN 4108-4 & ISO 12524

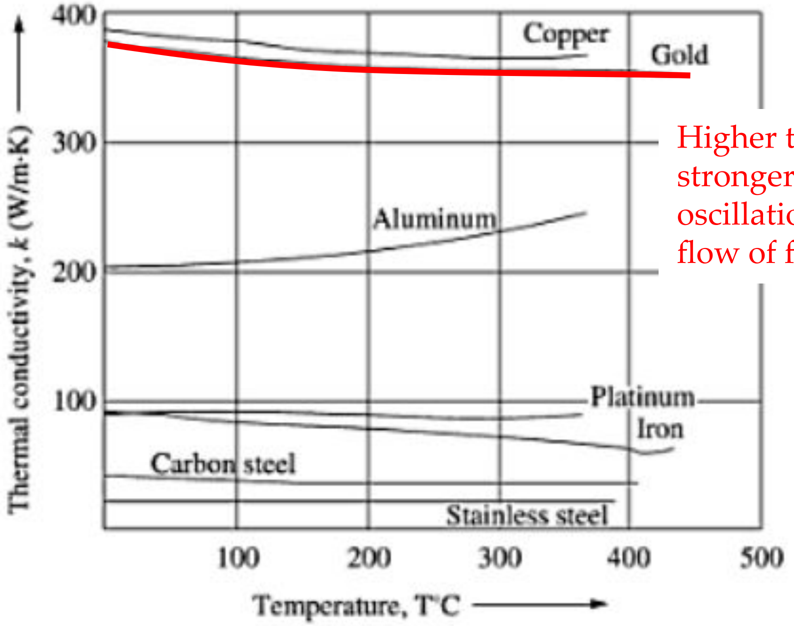
Gases and metals

- For metals increasing or decreasing effect of temperature level
- For gases increasing effect of temperature level

Thermal Conductivity and Temperature

Metals:

- Conductivity is sum of vibration transfer and free electron transfer
- Free electrons provide huge fraction of entire heat transfer
- Higher temperature causes higher lattice vibrations
- Higher lattice vibrations obstruct free electron transfer

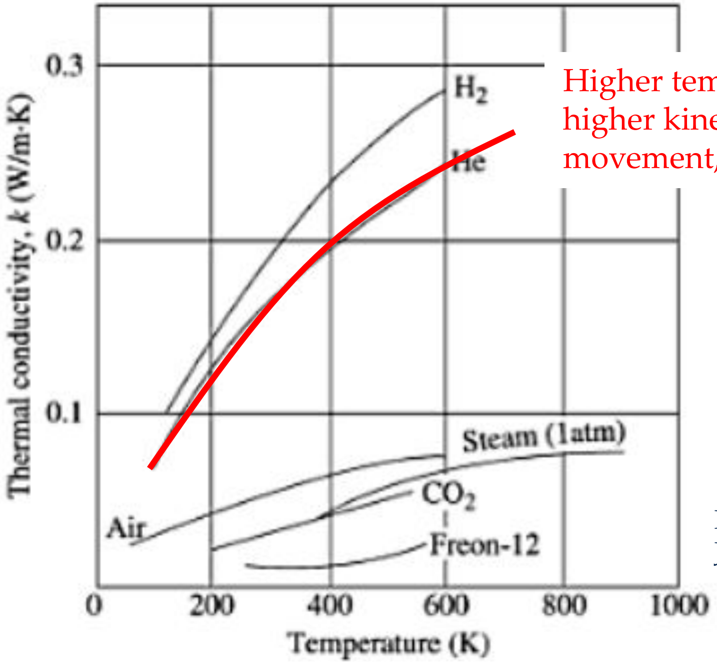


Higher temperature causes stronger lattice vibrations, oscillations obstruct flow of free electrons

Figure source: M.M.Rathore: Engineering Heat Transfer, Jones & Barlett Learning, 2011

Gases

- Molecules in continuous random motion
- Velocity of molecules increases with increasing temperature



Higher temperature causes higher kinetic energy, faster movement, higher impulse

Figure source: M.M.Rathore: Engineering Heat Transfer, Jones & Barlett Learning, 2011

TC and Heat Flow Direction

*An **isotropic material** is a material in which the thermal conductivity does not vary with the direction of heat flow.*

***Anisotropic materials** show a dependency of thermal conductivity on the heat flow direction.*

Examples are: wood, sedimentary rocks, metals that have undergone heavy cold pressing, fiber-reinforced composite structures.

Thermal Conductivity of Wood Depending on the Grain

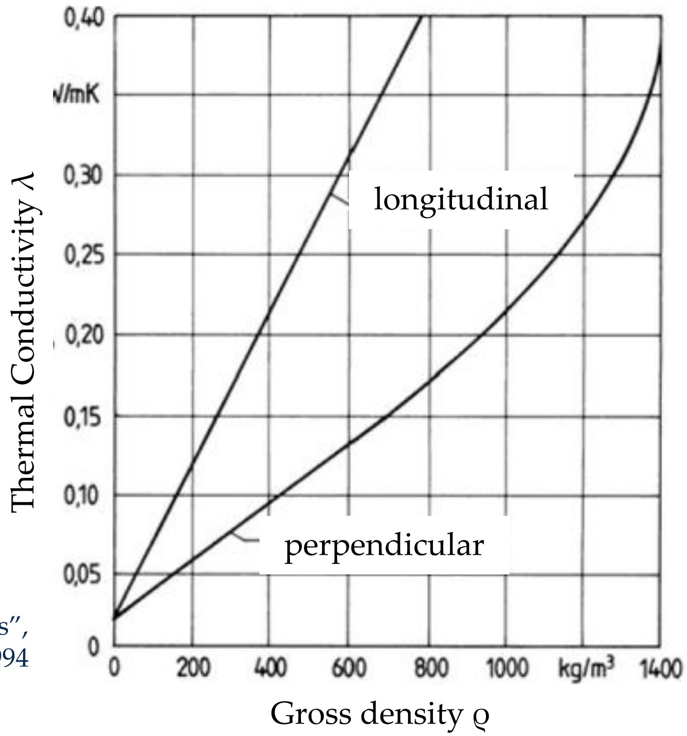


Figure source: H. Neuhaus: “Lehrbuch des Ingenieurholzbaus”, Springer Fachmedien Wiesbaden, 1994

*An **insulation material** shows an adverse dependency between thermal conductivity and gross density.*

Example mineral wool: thermal conductivity

Increases slightly if the gross density increases

For values above about 50 kg/m³.

BUT

The density increases strongly below a density

of 50 kg/m³.

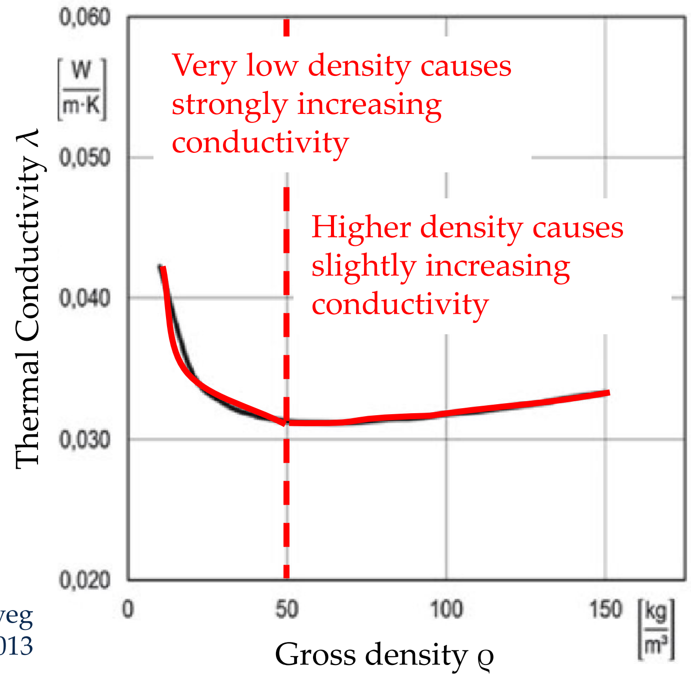
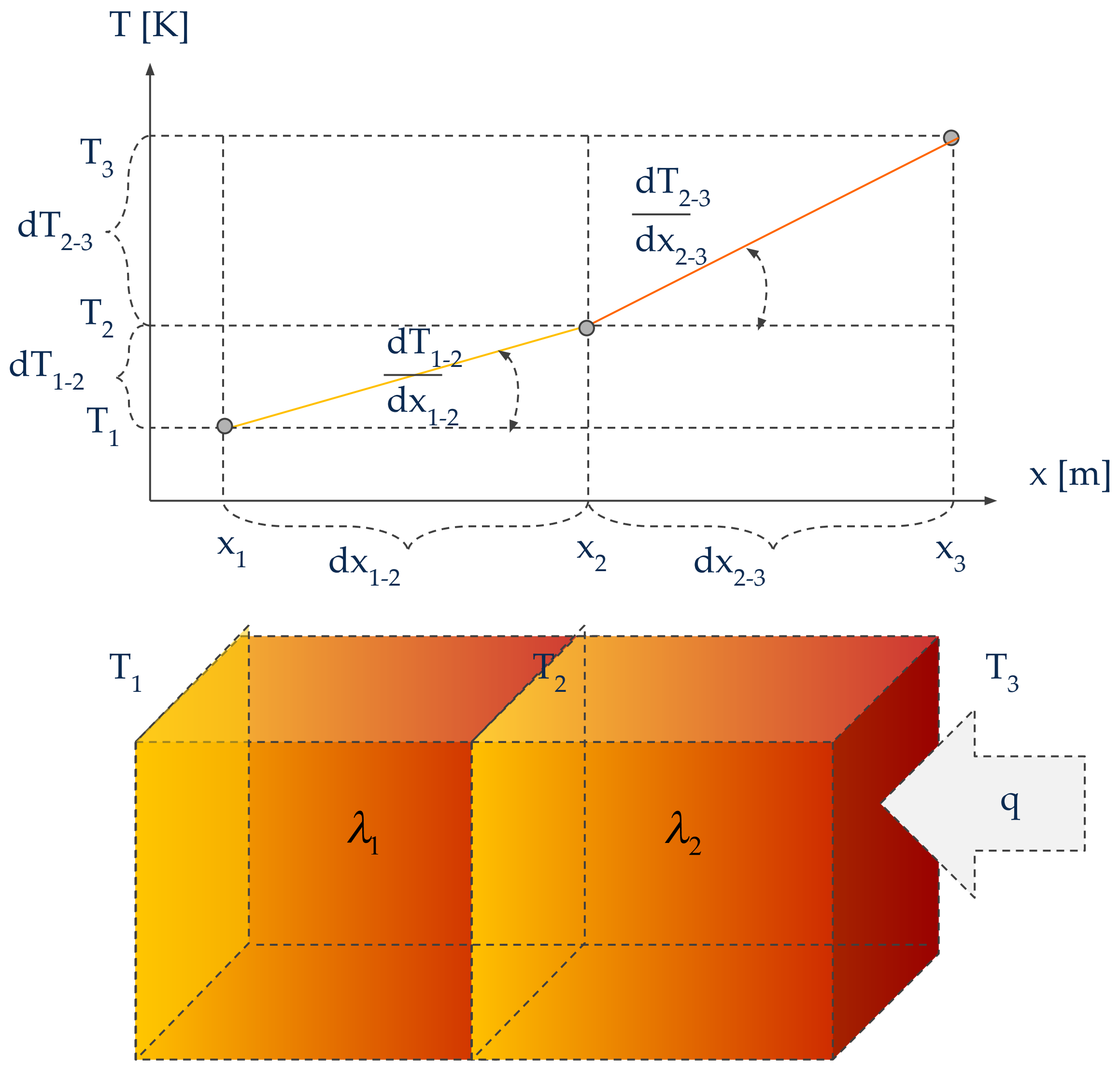


Figure source: W. M. Willems: "Lehrbuch der Bauphysik – Sch Wärme Feuchte Licht Brand Klima", 7. Auflage, Springer Vieweg Verglag, Wiesbaden, 2013

Heat Conduction 1D



Temperature gradients

$$\frac{dT_{1-2}}{dx_{1-2}} \; and \; \frac{dT_{2-3}}{dx_{2-3}} \left[\frac{W}{K}\right]$$

Thermal conductivities

$$\lambda_1 \; and \; \lambda_2 \left[\frac{W}{mK}\right]$$

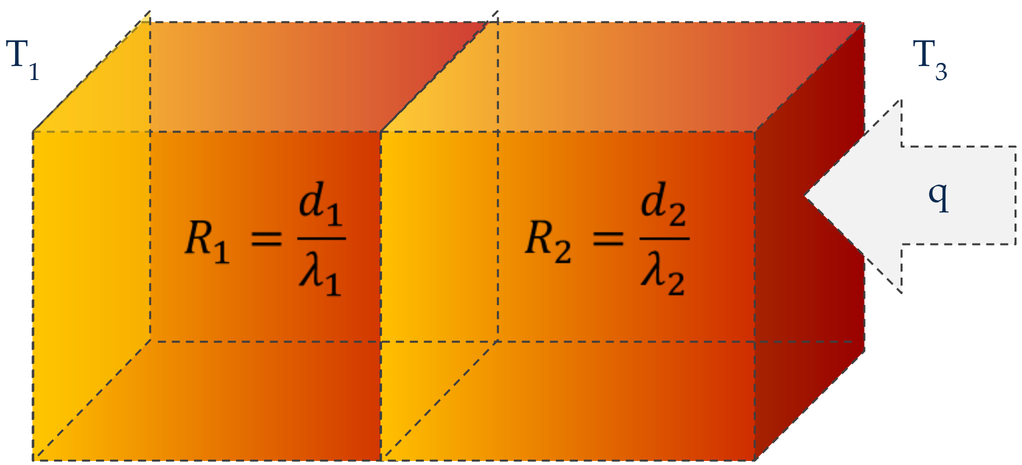
Resulting heat flux in each layer

$$q_1 = \left(\frac{dT_{1-2}}{dx_{1-2}} \cdot -\lambda_1\right)$$

$$q_2 = \left(\frac{dT_{2-3}}{dx_{2-3}} \cdot -\lambda_2\right)\left[\frac{W}{m^2}\right]$$

Resulting heat flux for the entire element:

$$q = dT_{1-3} \frac{1}{(R_1 + R_2)}$$



Entire temperature gradient

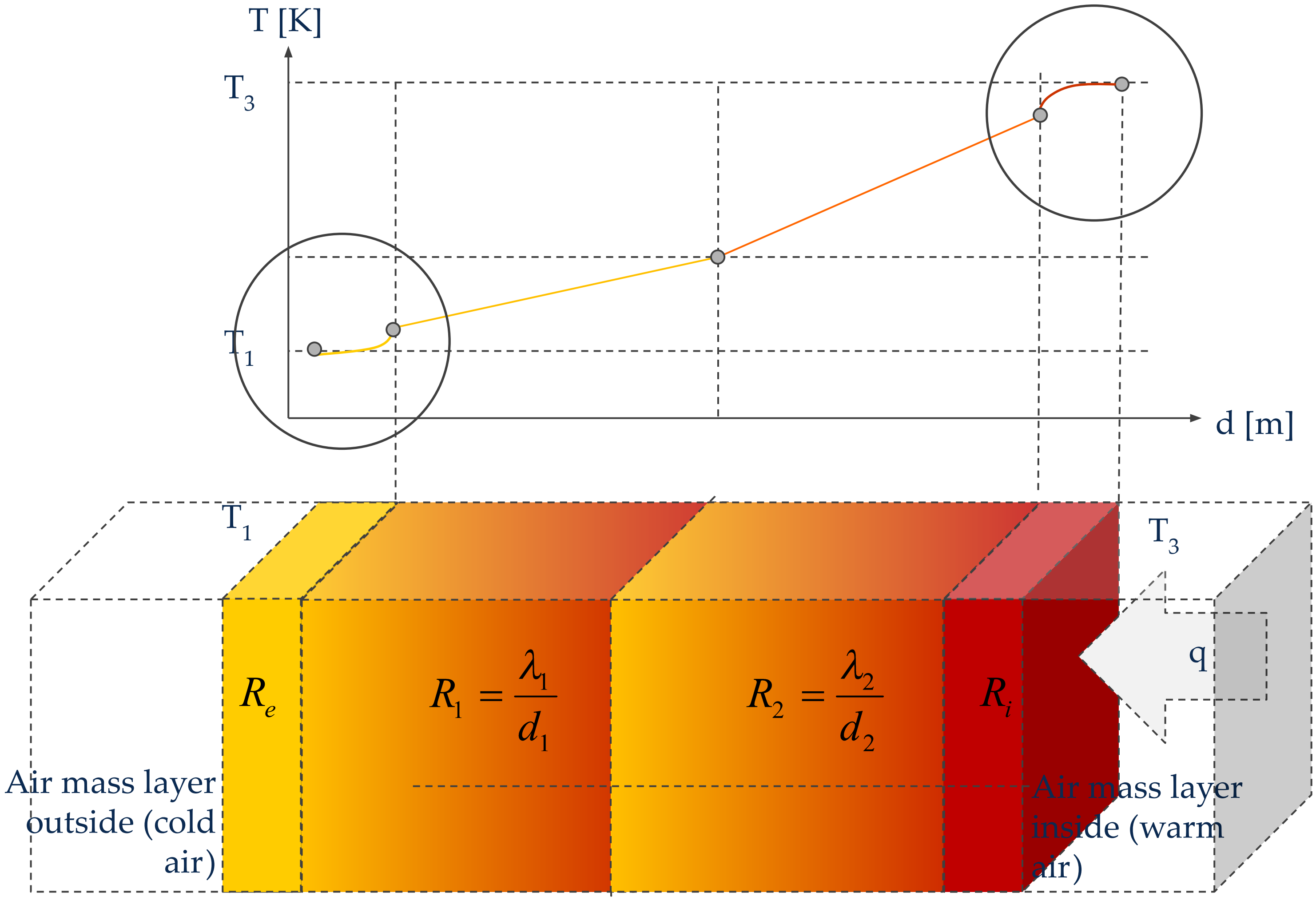
$$\frac{dT_{1-3}}{dx_{1-3}} \left[\frac{W}{K}\right]$$

Thermal conductivities

$$\lambda_1 \; and \; \lambda_2 \left[\frac{W}{mK}\right]$$

Entire heat flux

$$q = \frac{dT_{1-3}}{(R_1 + R_2)}$$



T [K]
T_3
T_1
d [m]
T_1
T_3
R_e
$R_1 = \frac{\lambda_1}{d_1}$
$R_2 = \frac{\lambda_2}{d_2}$
R_i
q
Air mass layer outside (cold air)
Air mass layer inside (warm air)

Heat conduction - Surface Resistance

Surface temperature of a building construction and air mass **temperature of** the adjacent **air mass** are **not equal** because:

- Energy is needed to transfer heat from solid state (solid envelope materials) to gaseous state (air mass)
- Convection processes of air masses influence transfer process
- Radiation exchange influences surface- near material layers

These influences can be summarized and simplified as an additional conduction resistance layer, the thermal **surface resistance**

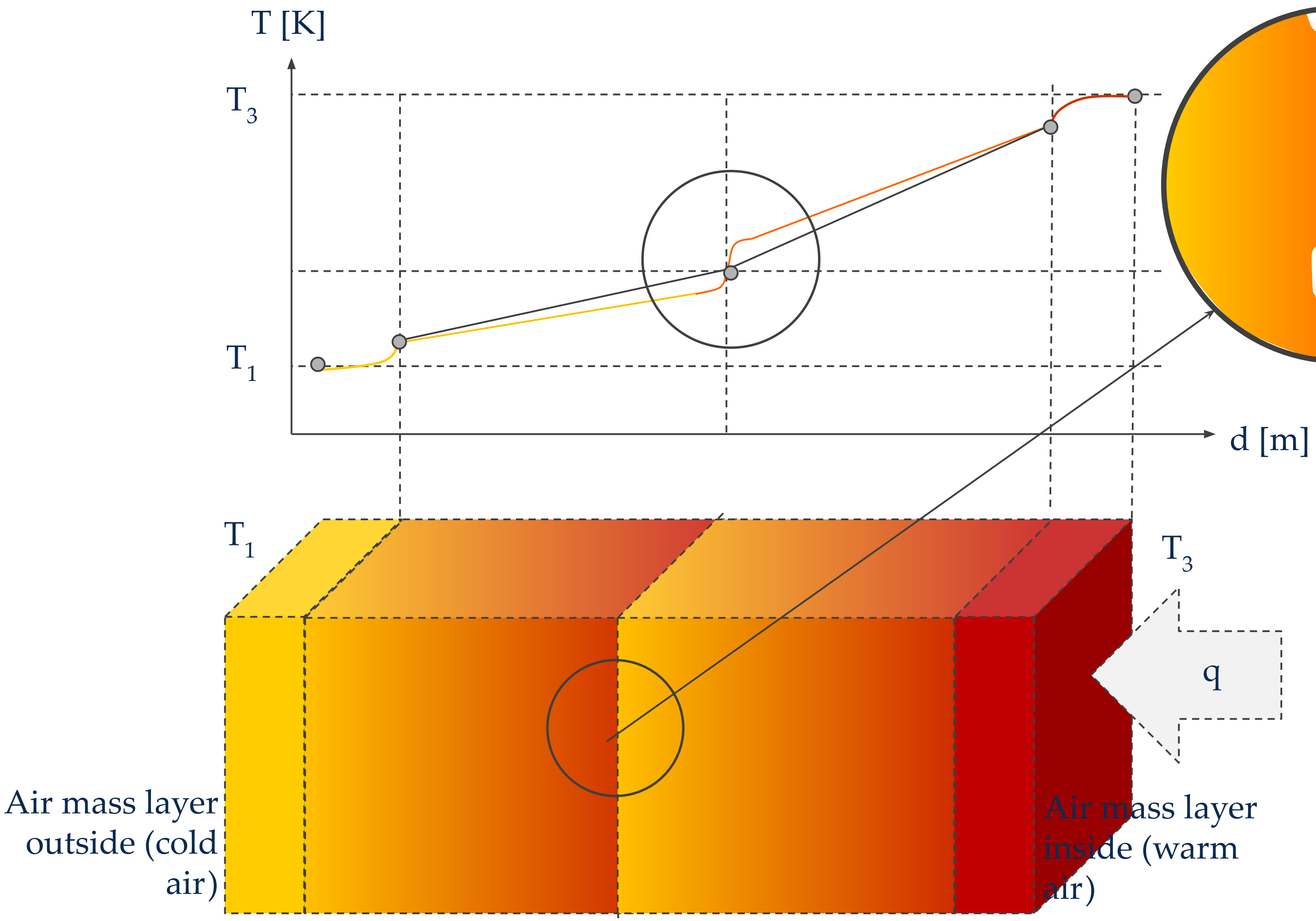
⇨ This resistance **is significant** and must be considered.
(Values are around 0.05 to 0.5 m^2K/W)

Heat conduction - Surface Resistance

Wind speed	Surface Position	Direction of heat flow	Resistance (m2.K/W)	
			Any other surface facing warm airspace	Bright Bradford foil facing the warm airspace
Still Air	Horizontal	Up	0.11	0.23
		Down	0.16	0.8
	45ºC slope	Up	0.11	0.24
		Down	0.13	0.39
	22.5ºC slope	Up	0.11	0.24
		Down	0.15	0.6
	Vertical	Horizontal	0.12	0.3
6.00 m/s (Summer)	Any position	Any direction	0.03	
3.00 m/s (Summer)	Any position	Any direction	0.04	
0.50 m/s (Internal air movement)	Any position	Any direction	0.08	

Figure source: www.bradfordinsulation.com.au

Heat Conduction - Contact Conditions



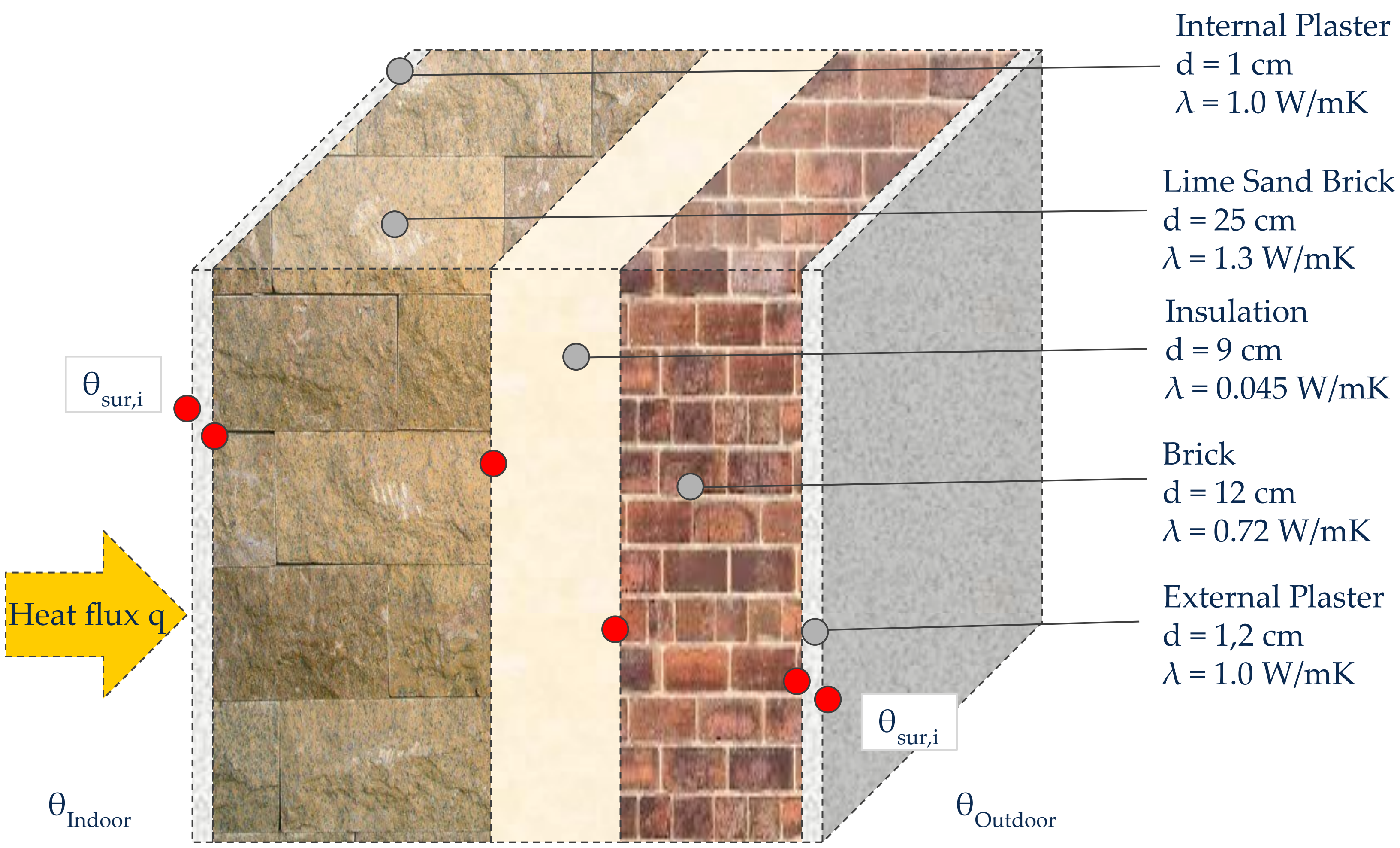
Interface temperature of one material layer and interface temperature **of the adjacent material layer** are **not equal** because:

- Most construction materials are rough and porous and are not in ideal contact with the adjacent material
- Interface contains several air gaps that serve as an additional insulation layer (low conductivity of air)

This effect can also be summarized and **simplified as** an additional conduction resistance layer, the **thermal contact resistance**

→ For building materials, contact resistance **can be neglected** (values are around 0.000005 to 0.0005 m^2K/W)

Example: 1D Steady-State



The heat flux is the same in all layers (steady state):

$$q = \frac{\Delta\theta_i}{R_{si}} \quad q = \frac{\Delta\theta_1}{R_{s1}} \quad q = \frac{\Delta\theta_2}{R_{s2}} \quad \dots \quad q = \frac{\Delta\theta_j}{R_{sj}} \quad \dots \quad q = \frac{\Delta\theta_n}{R_{sn}} \quad q = \frac{\Delta\theta_e}{R_{se}}$$

Rearranged as follows:

$$\Delta\theta_i = q \cdot R_{si} \quad \Delta\theta_1 = q \cdot R_{s1} \quad \Delta\theta_2 = q \cdot R_{s2} \dots \quad \Delta\theta_j = q \cdot R_{sj} \dots \quad \Delta\theta_n = q \cdot R_{sn} \quad \Delta\theta_e = q \cdot R_{se}$$

Temperature gradients between layers are added

$$\theta_i - \theta_e = \sum_{j=0}^{n+1} \Delta\theta_j = q \cdot \left(R_{si} + R_1 + R_2 + \dots + R_j + \dots + R_n + R_{se}\right)$$

Resulting heat flux for a multi-layer construction

$$q = \frac{\theta_i - \theta_e}{R_{si} + \sum_{j=1}^{n} R_j + R_{se}} \qquad R_{ges} = R_{si} + \sum_{j=1}^{n} R_j + R_{se}$$

Example: 1D Steady-State

Scheme for the calculation of the steady state temperature profile of a multilayered construction:

1. Calculation heat transmission resistances of all layers

$$R_j = \frac{d_j}{\lambda_j}$$

2. Calculation of total transmission resistance and U-value

$$R_{ges} = R_{si} + \sum_{j=1}^{n} R_j + R_{se} \qquad U = \frac{1}{R_{ges}}$$

3. Calculation of heat flux

$$q = U \left(\theta_{in} - \theta_{ex}\right)$$

4. Calculation of temperatures at each boundary

$$\theta_j = \theta_i - q \left(R_{si} + \sum_{k=1}^{j} R_k\right)$$

Example: 1D Steady-State

Construction layers	Thickness d in m	T. conductivity l in W/mK	Thermal resistance R in m^2K/W
Inner heat transfer resistance	---		R_{si}=0,13
Inner plaster	d_1 = 0,010	l_1 = 1,010	$R_1 = \frac{d_1}{\lambda_1}$ R_1=0,0099
Lime sand stone	d_2 = 0,250	l_2 = 1,320	$R_2 = \frac{d_2}{\lambda_2}$ R_2=0,1890
Mineral wool insulation	d_3 = 0,090	l_3 = 0,045	$R_3 = \frac{d_3}{\lambda_3}$ R_3=2,0000
Brick	d_4 = 0,120	l_4 = 0,720	$R_4 = \frac{d_4}{\lambda_4}$ R_4=0,1670
Outer plaster	d_5 = 0,012	l_5 = 1,100	$R_5 = \frac{d_5}{\lambda_5}$ R_5=0,0110
Outer heat transfer resistance	---		R_{se}=0,04

Heat transmission resistance $R = R_{si} + R_1 + R_2 + R_3 + R_4 + R_5 + R_{se}$

= 2,547 m^2K/W

U-value U = 1/R = 0,393 W/m^2K

Heat flux through the wall $q = U \cdot (\theta_i - \theta_e)$ = 9,816 W/m^2

Example: 1D Steady-State

Construction layers	Thermal resistance R in m^2K/W	$R_{1,j} = R_{si} + \sum_{i=1}^{i} R_i$	Temperatures θ in °C $\theta_{j,j+1} = \theta_i - q\left(R_{si} + \sum_{i=1}^{i} R_i\right)$
Inner heat transfer resistance	R_{si}=0,13	$R_{0,0}$=0,13	θ_i = 20°C θ_{si} = 18,72°C θ_{12} = 18,63°C θ_{23} = 16,77°C θ_{34} = -2,86°C θ_{45} = -4,5°C θ_{se} = -4,61°C θ_e = -5°C
Inner plaster	R_1=0,0099	$R_{0,1}$=0,1399	
Lime sand brick	R_2=0,1890	$R_{0,2}$=0,3289	
Mineral wool insulation	R_3=2,0000	$R_{0,3}$=2,3289	
Brick	R_4=0,1670	$R_{0,4}$=2,4959	
Outer plaster	R_5=0,0110	$R_{0,5}$=2,5069	
Outer heat transfer resistance	R_{se}=0,04	$R_{0,e}$=2,5469	
Heat flux through the wall		= 9,816 W/m^2	

Heat Conduction 2D Steady-State

In some cases, heat transfer must be seen two-dimensional steady-state:

Steady- state assumes

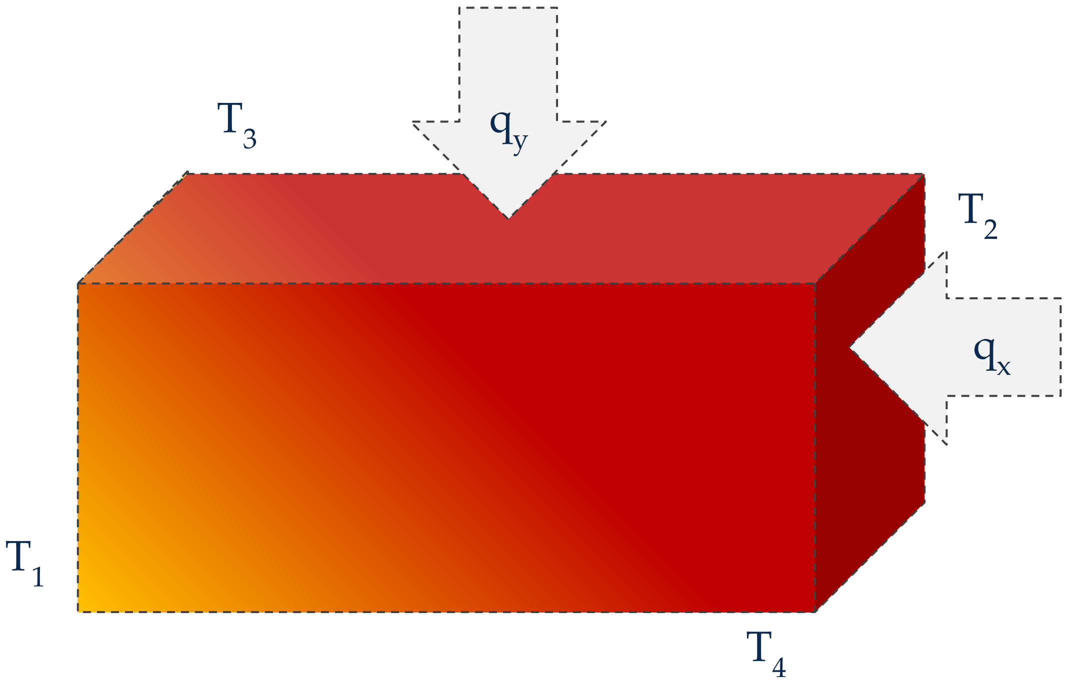
- Time- constant boundary conditions (indoor conditions, climate conditions, heat sources or sinks,…)
- And thus no relevance of thermal storage effects

Two- dimensional instead of one- dimensional which assumes

- Heat flux can be described in x- and y-direction
- Thus no relevance of three- dimensional effects

Solely **heat conduction related** which assumes

- Convection or radiation transfer can be neglected or described via thermal conductivity value (e.g. air layers)



Temperature gradient in x- direction

$$\frac{T_2 - T_1}{x_2 - x_1} = \frac{dT_x}{dx} \left[\frac{W}{K}\right]$$

Temperature gradient in y-direction

$$\frac{T_4 - T_3}{y_2 - y_1} = \frac{dT_y}{dy} \left[\frac{W}{K}\right]$$

Resulting heat flux Density in each direction

?

Heat Conduction- Simplified 2-D

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Analytical approaches only for **simple geometries** and boundary conditions

Complex geometries and boundary conditions can be handled with **numerical methods**, which

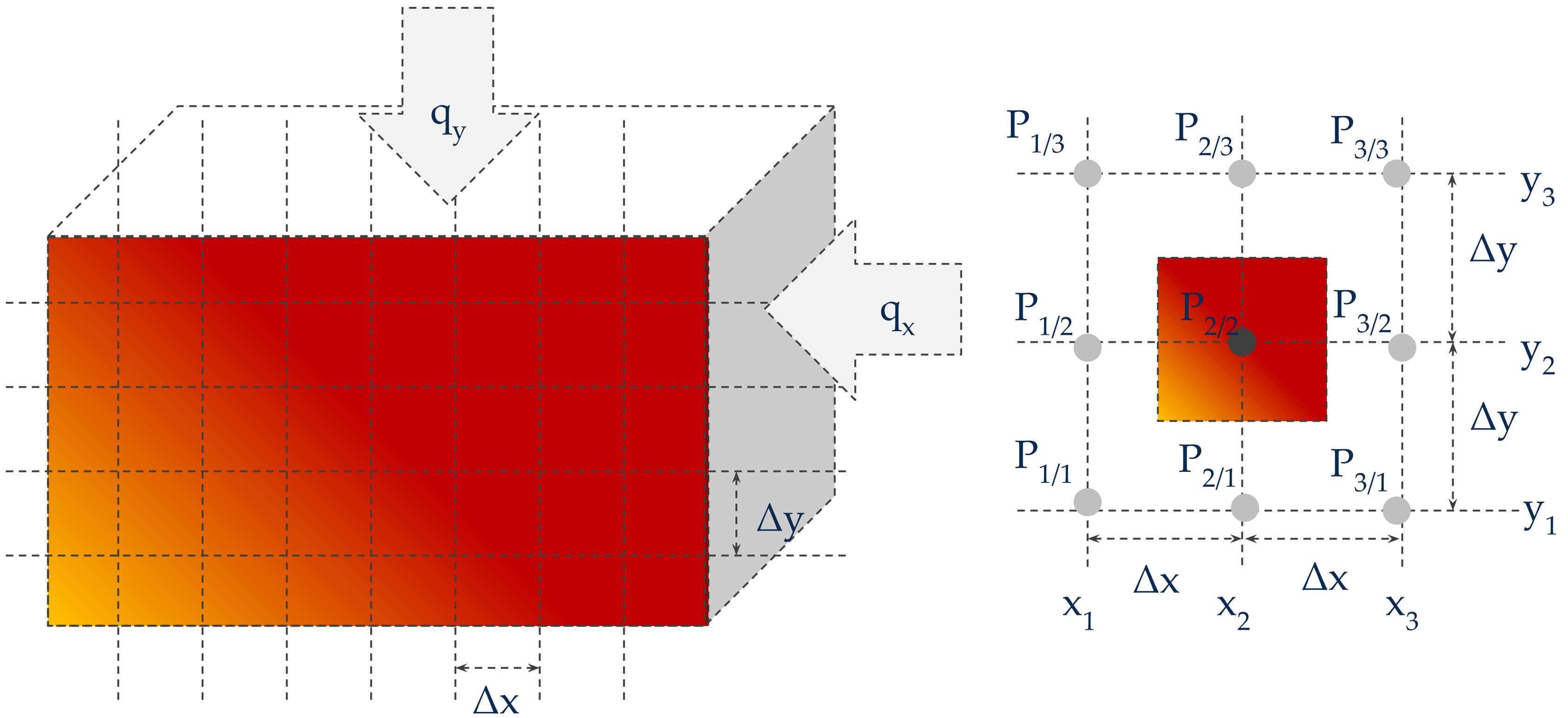
- Provide solutions for **discrete points**
- Give only an **approximation**

Two established numerical methods are

- **Finite Difference Method**
- **Finite Element Method**

Finite- Difference- Method – 2d Steady St.

- Each differential equation is approximated by a finite linear difference equation
- Construction is divided into equal segments of Δx an Δy
- Nodal points $P_{x/y}$ represent surrounding area of the size Δx and Δy

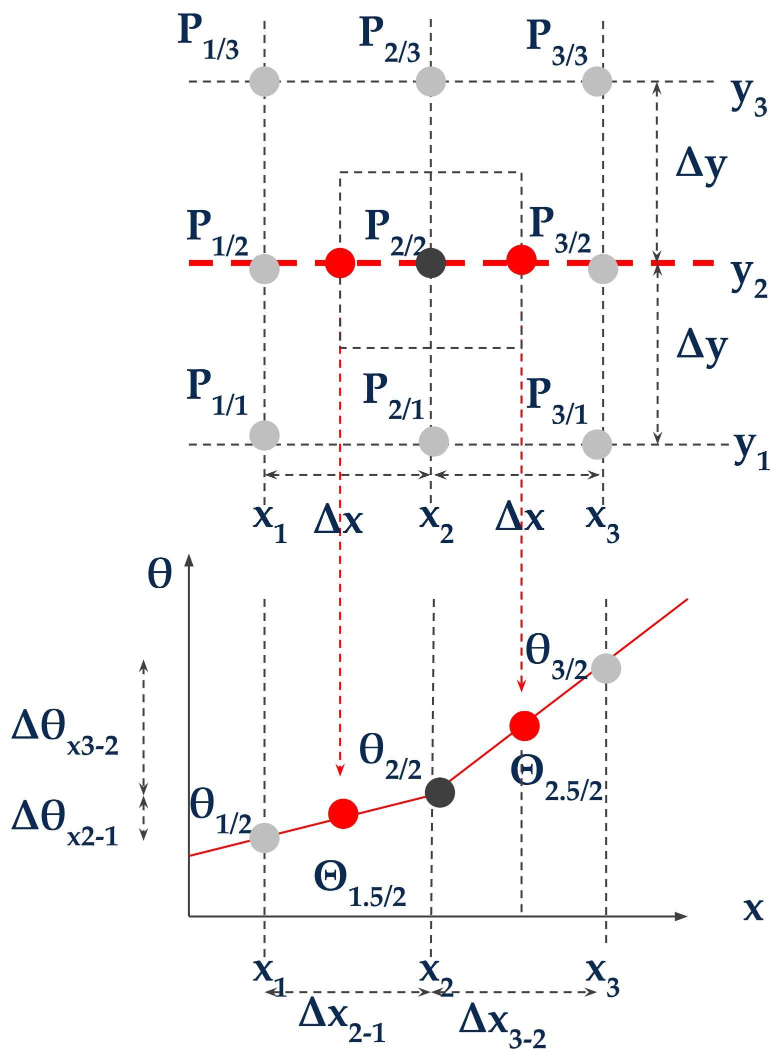


Temperature gradients in x- direction can be transferred into finite difference form

Equations for nodes in x-direction are:

$$\left.\frac{\partial\theta}{\partial x}\right|_{P1.5/2} \approx \frac{\theta_{2/2}-\theta_{1/2}}{x_2-x_1} = \frac{\Delta\theta_{x2-1}}{\Delta x_{2-1}}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{P2.5/2} \approx \frac{\theta_{3/2}-\theta_{2/2}}{x_3-x_2} = \frac{\Delta\theta_{x3-2}}{\Delta x_{3-2}}$$

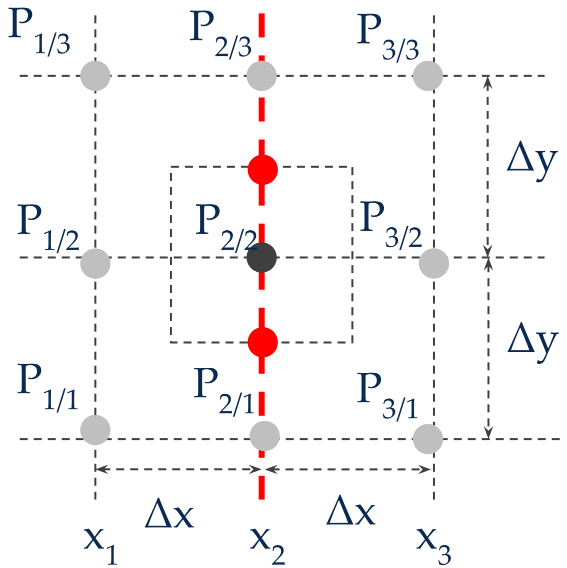


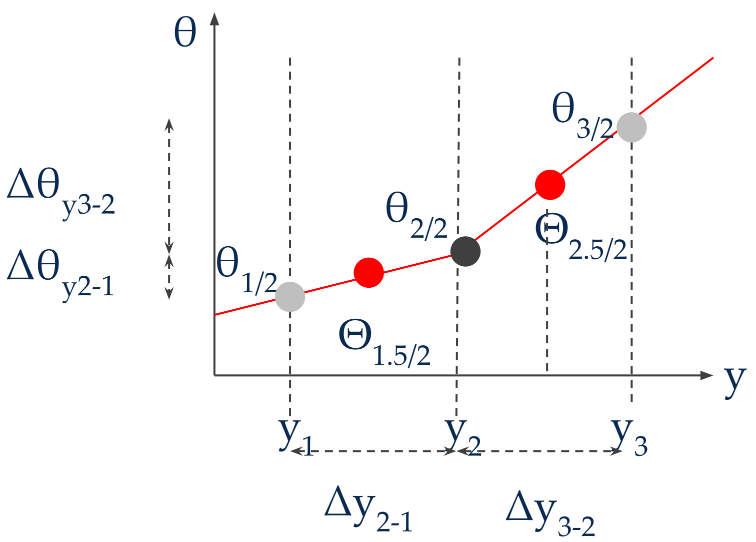
Temperature gradients in y- direction can be transferred into finite difference form in the same way

$$\left.\frac{\partial\theta}{\partial y}\right|_{P1.5/2} \approx \frac{\theta_{2/2}-\theta_{2/1}}{y_2-y_1} = \frac{\Delta\theta_{x2-1}}{\Delta y_{2-1}}$$

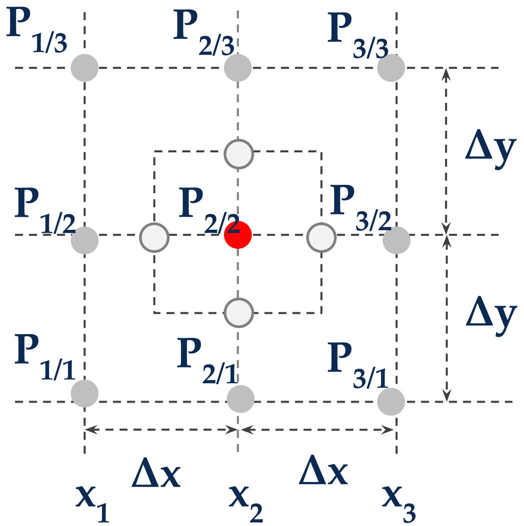
Equation for red points on y-axis are:

$$\left.\frac{\partial\theta}{\partial y}\right|_{P2.5/2} \approx \frac{\theta_{2/3}-\theta_{2/2}}{y_3-y_2} = \frac{\Delta\theta_{y3-2}}{\Delta y_{3-2}}$$





Temperature gradient for current (red marked) node is consequently given as:



$$\left.\frac{\partial\theta}{\partial x}\right|_{P2/2} \approx \frac{\left.\frac{\partial\theta}{\partial x}\right|_{P1.5/2} - \left.\frac{\partial\theta}{\partial x}\right|_{P2.5/2}}{dx} = \frac{\left(\frac{\theta_{2/2}-\theta_{1/2}}{dx}\right)-\left(\frac{\theta_{3/2}-\theta_{2/2}}{dx}\right)}{dx} = \frac{2\cdot\theta_{2/2}-\theta_{1/2}-\theta_{3/2}}{dx^2}$$

$$\left.\frac{\partial\theta}{\partial y}\right|_{P2/2} \approx \frac{\left.\frac{\partial\theta}{\partial y}\right|_{P2/1.5} - \left.\frac{\partial\theta}{\partial y}\right|_{P2/2.5}}{dy} = \frac{\left(\frac{\theta_{2/2}-\theta_{2/1}}{dy}\right)-\left(\frac{\theta_{2/3}-\theta_{2/2}}{dy}\right)}{dy} = \frac{2\cdot\theta_{2/2}-\theta_{2/1}-\theta_{2/3}}{dy^2}$$

Applying the 2-dimensional Laplace equation (difference of temperature gradient change in both heat flux directions is zero in steady- state case):

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{\partial^2 \theta}{\partial y^2} = 0$$

Gives this FD solution:

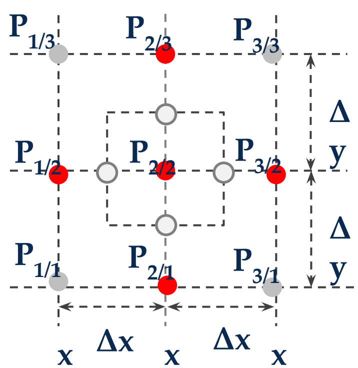
$$\frac{2 \cdot \theta_{2/2} - \theta_{1/2} - \theta_{3/2}}{dx^2} - \frac{2 \cdot \theta_{2/2} - \theta_{2/1} - \theta_{2/3}}{dy^2} = 0$$

Which can be reduced to an approximate algebraic equation if dx and dy are equal:

$$4 \cdot \theta_{2/2} - \theta_{1/2} - \theta_{3/2} - \theta_{2/1} - \theta_{2/3} = 0$$

$$\theta_{2/2} = \frac{\theta_{1/2} + \theta_{3/2} + \theta_{2/1} + \theta_{2/3}}{4}$$

For given example: temperature at nodal point equals arithmetic average of the four adjacent nodes

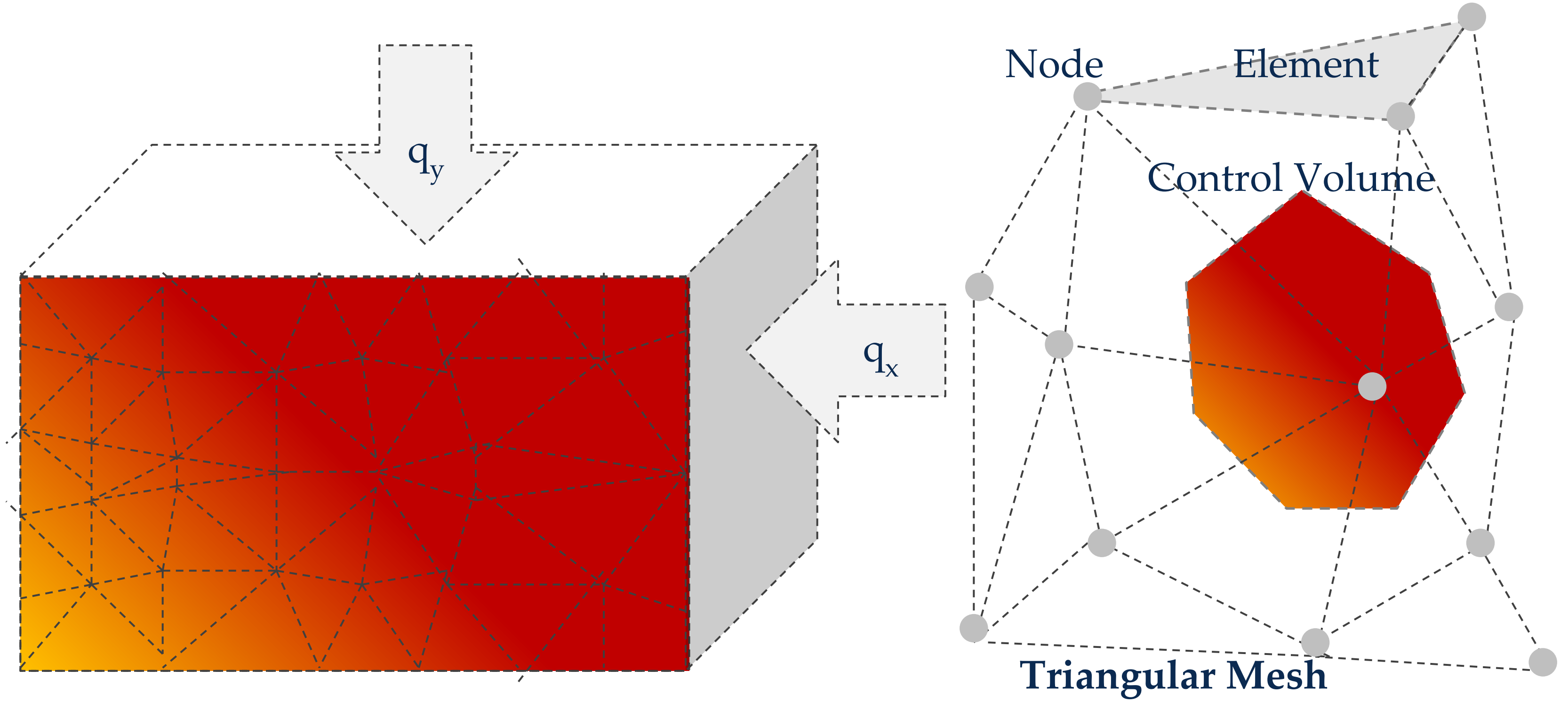


For the example of **control volume method**

Shape functions describe change of state in each volume element

Construction is divided by a (triangular) mesh

Control volumes are generated by connecting the centers of adjacent elements



Example for two- dimensional heat conduction under steady state conditions:

$$\lambda\left(\frac{\partial^2\theta}{\partial x^2}-\frac{\partial^2\theta}{\partial y^2}\right)=0$$

Key step is integration of general 2D SS heat conduction over two- dimensional control volume:

$$\int\left(\lambda\left(\frac{\partial^2\theta}{\partial x^2}-\frac{\partial^2\theta}{\partial y^2}\right)\right)dV=0$$

Volume integrals are rewritten as integrals over entire bounding surface of the control volume by using Gauss divergence theorem

Direction cosine of unit vector of bounding surface in x- direction

$$\int\left(\left(\lambda\left(\frac{\partial\theta}{\partial x}\right)n_x-\lambda\left(\frac{\partial\theta}{\partial y}\right)n_y\right)dA+QdV\right)=0$$

Direction cosine of unit vector of bounding surface in y- direction

Surface Integral can be evaluated by midpoint rule as:

$$\int \left(\left(\lambda \left(\frac{\partial \theta}{\partial x} \right) n_x - \lambda \left(\frac{\partial \theta}{\partial y} \right) n_y \right) dA \right) \approx \sum_{i=1}^{NI} \left(\left(\lambda \frac{\partial \theta}{\partial x} n_x - \lambda \frac{\partial \theta}{\partial y} n_y \right) \Delta A \right)$$

Number of faces (1 to NI)

Face area = portion of entire surface around control volume

Temperature gradients can be calculated via shape functions N_x of linear triangular elements:

Shape Functions

$$\frac{\partial \theta}{\partial x} = \sum_{k=1}^{3} \frac{\partial N_k}{\partial x} \theta_k \quad and \quad \frac{\partial \theta}{\partial y} = \sum_{k=1}^{3} \frac{\partial N_k}{\partial y} \theta_k$$

Discretized equation for temperature of all control volumes is then given as:

$$\sum_{i=1}^{NI} \left(\left(\lambda \left(\sum_{k=1}^{3} \frac{\partial N_k}{\partial x} \theta_k \right) n_x - \lambda \left(\sum_{k=1}^{3} \frac{\partial N_k}{\partial y} \theta_k \right) n_y \right) \Delta A \right)$$

Resulting system of algebraic equations is incorporated with boundary conditions into numerical solver

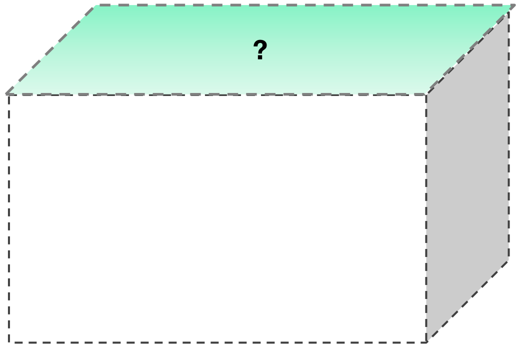
Boundary Conditions

Numerical **solutions of heat conduction problems** offer methods to estimate temperature distribution within an area or volume, **depending on boundary conditions** (steady state or transient)

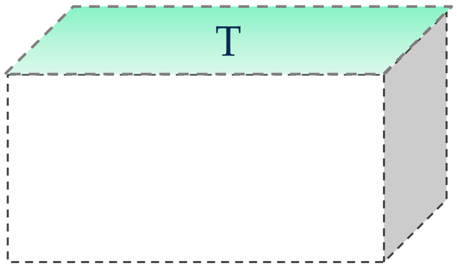
Appropriate solution can only be found if boundary conditions are known

There are three common types of boundary conditions:

1. Defined surface temperatures
2. Defined surface heat flux
3. Specified surface heat balance



Dirichlet boundary condition



Called boundary condition of first type

Temperature at the boundary of the body is given as a function of time and position or simply as a constant value

Description as: $\theta(x,t) = f(x,t)$

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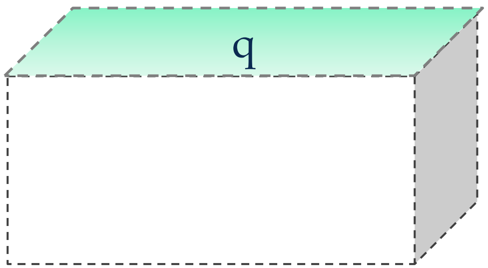
Neumann boundary condition



Called boundary condition of second type

Heat flux q at the boundary of the body is given

Description as:

$$q(x,t) = -\lambda \frac{\partial \theta}{\partial n}(x,t)$$

Allows us to determine the partial derivative of the temperature with respect to outward normal vector n:

$$\frac{\partial \theta}{\partial n}(x,t) = -\frac{q(x,t)}{\lambda}$$

Special case: adiabatic boundary conditions:

$$\frac{\partial \theta}{\partial n}(x,t) = 0$$

Cauchy boundary condition

Called boundary condition of third type

Describes correlation between temperature value and derivative of solution

Assumption: heat flux into body surface must equal heat flux out of the body surface (d=0, C=0)

For convective transfer given as:

Convective heat transfer coefficient

$$-\lambda \frac{\partial \theta}{\partial n}(x,t) = \alpha_{conv} \cdot (\theta(x,t) - \theta_f)$$

Temperature of the surrounding fluid (also called T_infinity)

For radiant transfer:

$$-\lambda \frac{\partial \theta}{\partial n}(x,t) = \varepsilon \cdot \sigma_{StBo} \cdot (T(x,t)^4 - T_{Surr}^{\ 4}) \approx \alpha_{rad} \cdot (\theta(x,t) - \theta_f)$$

Combined:

$$-\lambda \frac{\partial \theta}{\partial n}(x,t) = (\alpha_{conv} + \alpha_{rad}) \cdot (\theta(x,t) - \theta_f)$$

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Advanced Computational and Civil Engineering Structural Studies

Exercise 1 Therm (LBNL)

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Dresden, 18.04.2019

Therm - overview

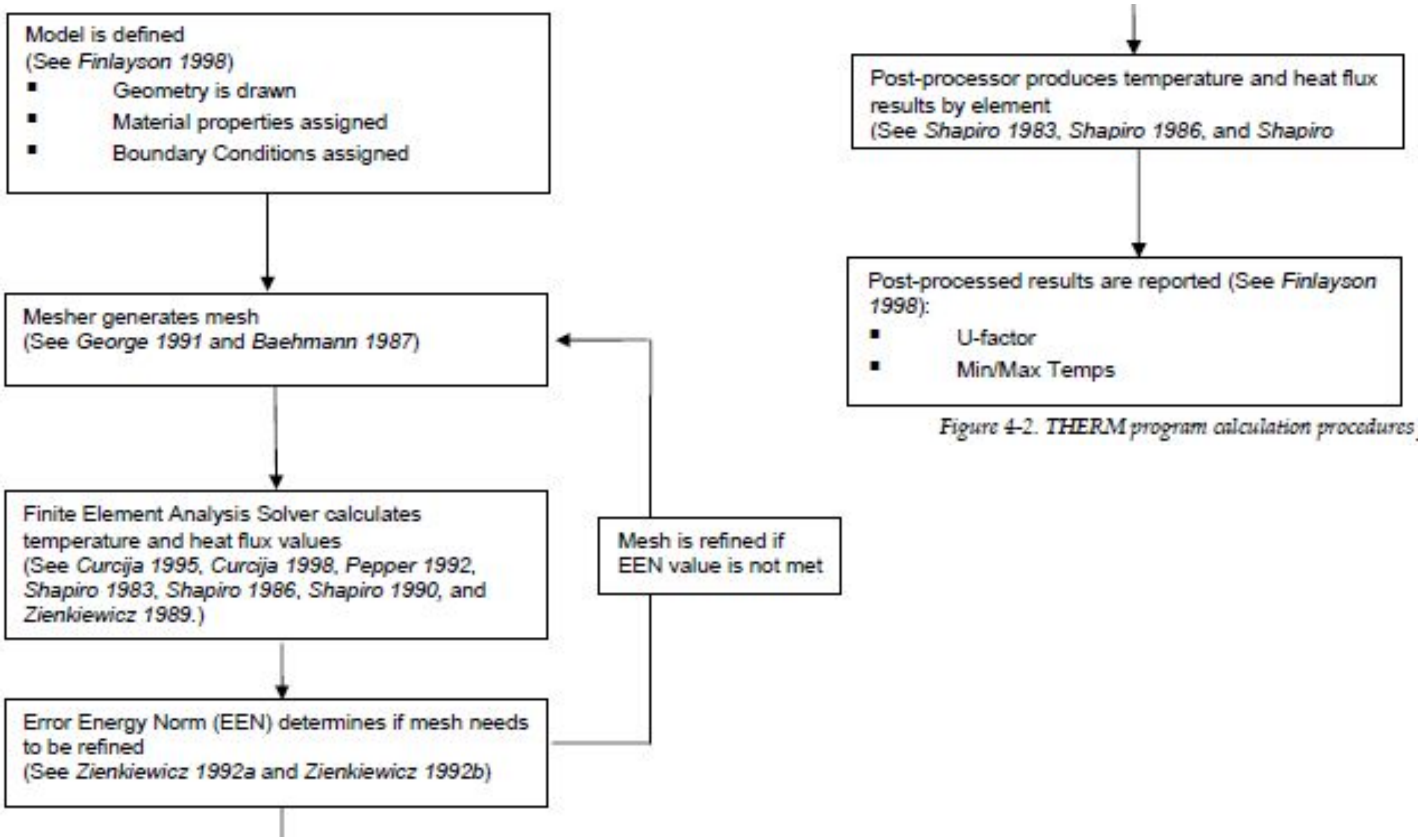


Figure 4-2. THERM program calculation procedures flow chart.

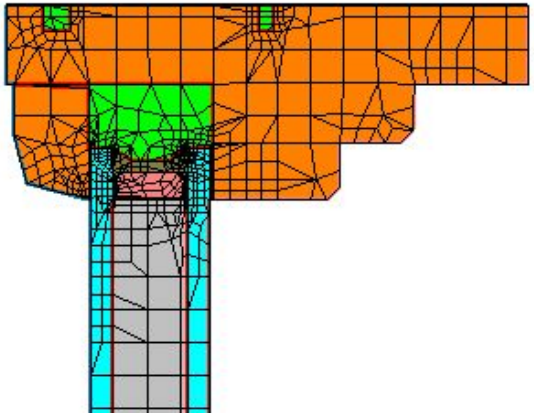
Therm – Mesh Generation

Made up of a finite number of non- overlapping subregions that cover the whole region

Well conditioned finite element mesh requires sophisticated FEM solution method knowledge, therefore:

THERM provides automatic mesh generation via Finite Quadtree- algorithm:

Object domain is divided into set of squares (hierarchic quadrants tree)



Subdivision is performed until:

- Each quadrant contains only one material
- Size difference between adjacent elements is balanced
- Entire domain is subdivided into triangles resp. quadrilaterals

Therm – Finite Element Method

Finite element solver: CONRAD

Derived from public- domain computer programs TOPAZ2D and FACET

Method assumes constant boundary conditions (steady- state) and physical properties

Governing partial differential equation for two- dimensional heat conduction including internal heat generation:

$$\lambda \cdot \left(\frac{\partial^2 \theta}{\partial x^2} - \frac{\partial^2 \theta}{\partial y^2} \right) + q_{\text{int}} = 0$$

Finite- element analysis is based on method of weighted residuals (Galerkin form which relies on algebraic shape functions)

Therm – Exercise 1D

in

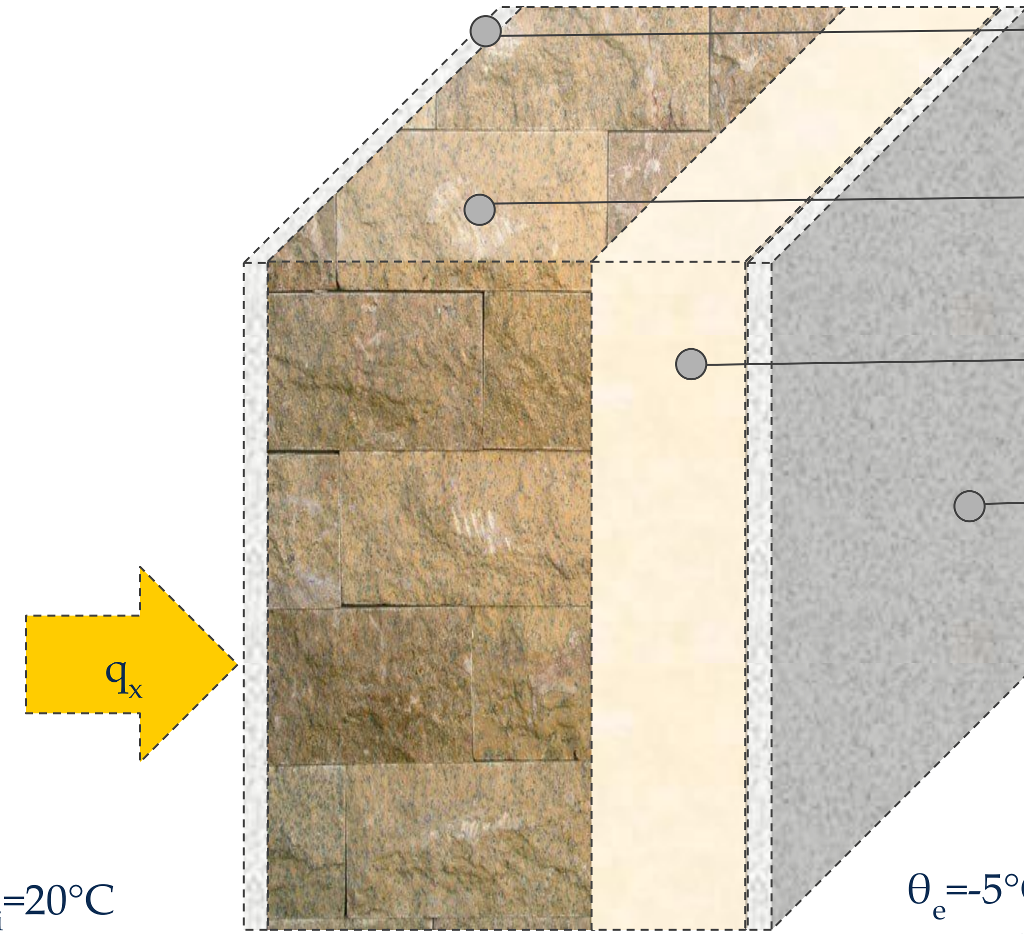
Given the wall example you should solve the following tasks:

1. Model this wall example geometry in THERM
2. Create all materials which are needed for this construction in THERM material library
3. Create all boundary conditions which are needed for this construction in THERM boundary conditions library
4. Assign materials and create (F10) and assign boundary conditions to the modeled construction
5. Create U-Factor name for y- direction and assign it by double clicking the created boundary conditions (vertial inside)
6. Run the calculation (F9)
7. Check your resulting U-Factor
8. Get the U-Value for this construction via manual calculation
9. Compare both U-Values

Compute the following tasks for the same external wall example as an edge

1. Model wall example geometry in THERM: notice that minimum distance from end to inner edge is 1m
2. Assign materials and create (F10) and assign boundary conditions to the modeled construction
3. Create U-Factor names for x- direction and assign it to horizontal element, do the same for U-Factor name in y-direction
4. Run the calculation (F9)
5. Check your resulting U-Factor in x- and in y-direction
6. Compare these values with your manual calculation result

Example: multilayer construction



Internal Plaster
d = 1 cm
λ = 1.0 W/mK

Lime Sand Brick
d = 25 cm
λ = 1.3 W/mK

Insulation
d = 9 cm
λ = 0.045 W/mK

External Plaster
d = 1 cm
λ = 1.0 W/mK

θ_i=20°C
α_i=20 W/m²K

θ_e=-5°C
α_e=8 W/m²K