Квазистационарное приближение.

$$\left|\frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}\right| \ll \left|\lambda(\vec{E}+\vec{E}_{\text{стор}})\right|. \quad \operatorname{rot}\vec{H} = \frac{4\pi}{c}\vec{j},$$

$$\operatorname{rot}\operatorname{rot}\mathbf{H} = -\Delta\mathbf{H},$$

$$\Delta\vec{H} = \frac{4\pi\lambda\mu}{c^2}\frac{\partial\vec{H}}{\partial t}. \quad \operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t},$$

$$\operatorname{div}\vec{B} = 0,$$

$$\Delta\vec{E} = \frac{4\pi\lambda\mu}{c^2}\frac{\partial\vec{E}}{\partial t}. \quad \operatorname{div}\vec{D} = 4\pi\rho.$$

$$\vec{D} = \varepsilon\vec{E}, \quad \vec{B} = \mu\vec{H}, \quad \vec{j} = \lambda\vec{E}.$$

$$\Delta\vec{H} = -\frac{4\pi\lambda}{c}\operatorname{rot}\vec{E} + \vec{\nabla}\operatorname{div}\vec{H}.$$

$$\Delta \vec{j} = \frac{4\pi\lambda\mu}{c^2} \frac{\partial \vec{j}}{\partial t} \cdot \quad \vec{j} = \frac{c}{4\pi} \operatorname{rot} \vec{H},$$

$$\vec{E} = \frac{\vec{j}}{\lambda}$$

Проникновение периодически меняющихся полей в проводник (в квазистационарном приближении). Скин-эффект.

$$\vec{H} = \vec{H}_0 e^{-i\omega t} \quad B_n^{I} = B_n^{II}, \quad H_\tau^{I} = H_\tau^{II}$$

$$\Delta \vec{H} = \frac{4\pi\lambda\mu}{c^2}\frac{\partial \vec{H}}{\partial t} \rightarrow \vec{H} = H(y)\, e^{-i\omega t}\, \vec{e}_x .$$

$$\frac{d^2 H(y)}{dy^2} = -\frac{4\pi\mu\lambda i\omega}{c^2} H(y)$$

$$H(0) = H_0 . \qquad H(y) = e^{\alpha y} .$$

$$\alpha^2 = -\frac{4\pi i\lambda\mu\omega}{c^2} = -\frac{2i}{\delta^2}.$$

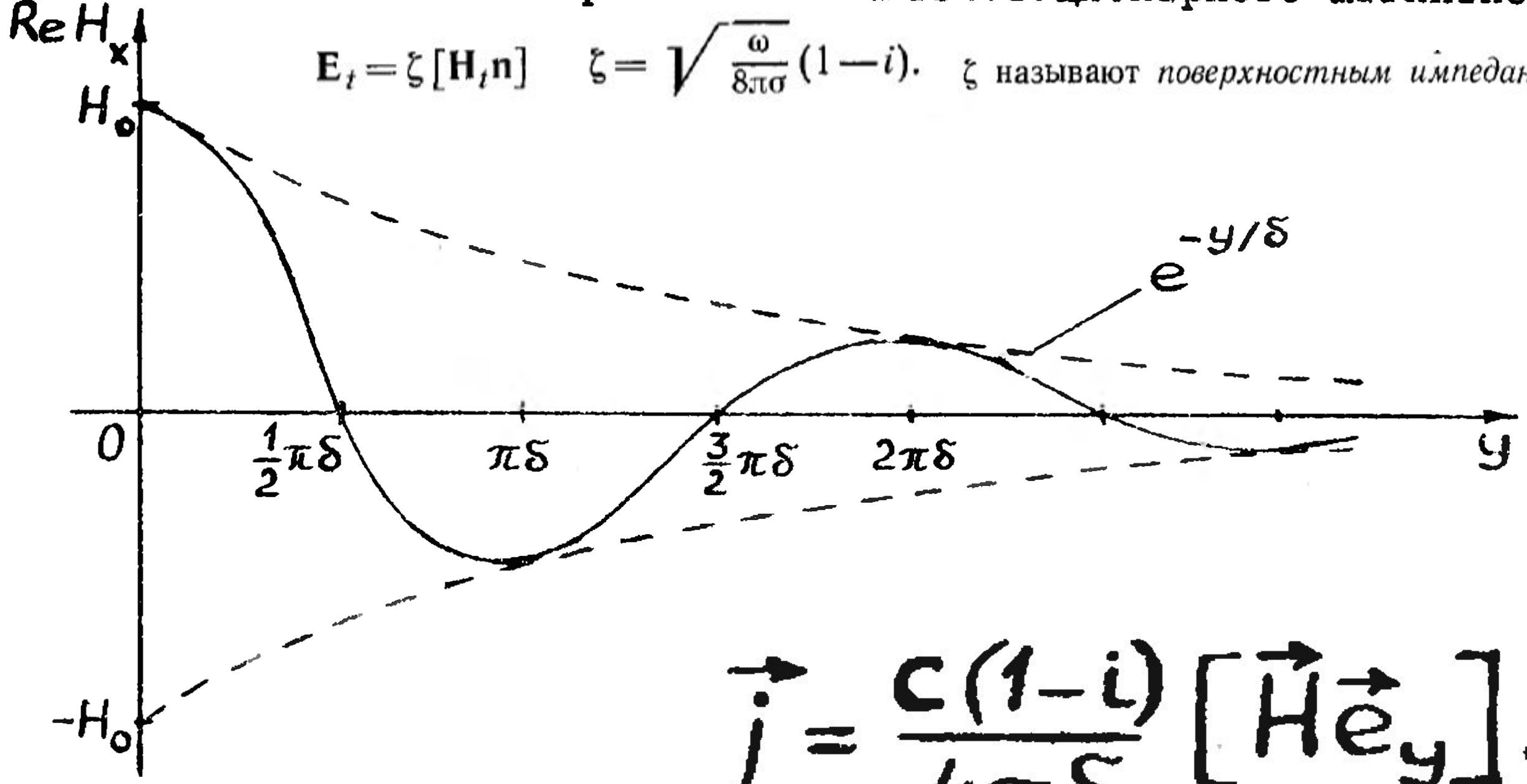
$$\delta = \frac{c}{\sqrt{2\pi\mu\lambda\omega}}$$

$$H(y) = C_1 \exp\left[\frac{(1-i)}{\delta}y\right] + C_2 \exp\left[-\frac{(1-i)}{\delta}y\right].$$

$$H(y) = H_0 \exp\left[-\frac{(1-i)}{\delta}y\right].$$

$$\vec{H} = \vec{H}_0 e^{-i(\omega t - \frac{y}{\delta})} e^{-y/\delta}.$$

$\mathbf{E}_t = \zeta[\mathbf{H}_t \mathbf{n}] \quad \zeta = \sqrt{\frac{\omega}{8\pi\sigma}}(1-i).$ ζ называют *поверхностным импедансом* проводника



$$\vec{j} = \frac{c(1-i)}{4\pi\delta}[\vec{H}\vec{e}_y].$$

$$\mathbf{E} = \sqrt{\frac{\omega}{8\pi\sigma}}(1-i)[\mathbf{H}\mathbf{n}].$$

$$|\vec{E}| \doteq \frac{\sqrt{2}\,c}{4\pi\lambda\delta}|\vec{H}|, \qquad |\vec{j}| = \frac{\sqrt{2}\,c}{4\pi\delta}|\vec{H}|,$$

Комплексное сопротивление

$$\mathscr{E}(t) = RJ(t), \qquad \mathscr{E} = \hat{Z}J \quad \Rightarrow \quad \mathscr{E} = Z(\omega)\,J. \qquad Z = R - \frac{i}{c^2}\,\omega L.$$

$$\frac{L}{c^2}\frac{d^2e}{dt^2} + R\frac{de}{dt} + \frac{e}{C} = \mathscr{E}. \quad \Rightarrow \quad Z = R - i\left(\frac{\omega L}{c^2} - \frac{1}{\omega C}\right).$$

Если же $\mathscr{E} = 0$, то ток в цепи представляет собой свободные электрические колебания. Частота (комплексная) этих колебаний определяется условием $Z = 0$, откуда

$$\omega = -i\,\frac{Rc^2}{2L} \pm \sqrt{\frac{c^2}{LC} - \left(\frac{Rc^2}{2L}\right)^2}$$

$$R \to 0 \qquad \qquad \omega = \frac{c}{\sqrt{LC}}$$

$$f_{ik} = \begin{pmatrix} 0 & e_x & e_y & e_z \\ -e_x & 0 & -h_z & h_y \\ -e_y & h_z & 0 & -h_x \\ -e_z & -h_y & h_x & 0 \end{pmatrix}$$

$$j^i = \{ j^0 = c\rho, \vec{j} \}$$

$$\frac{\partial f^{ik}}{\partial x^k} = -\frac{4\pi}{c} j^i,$$

$$\frac{\partial j^k}{\partial x^k} = 0$$

$$\frac{\partial f_{ik}}{\partial x^m} + \frac{\partial f_{km}}{\partial x^i} + \frac{\partial f_{mi}}{\partial x^k} = 0$$

$$\frac{\partial F_{ik}}{\partial x^m} + \frac{\partial F_{km}}{\partial x^i} + \frac{\partial F_{mi}}{\partial x^k} = 0,$$

$$F_{ik} = \langle f_{ik} \rangle$$

$$F_{ik} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}.$$

$$\operatorname{rot} \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$

$$\operatorname{div} \vec{B} = 0.$$

$$j^i_{\text{полн}} = j^i_{\text{своб}} + j^i_{\text{связ}}$$

$$\frac{\partial j^k_{\text{своб}}}{\partial x^k} = \frac{\partial j^k_{\text{связ}}}{\partial x^k} = 0$$

$$\langle j^k_{\text{полн}} \rangle \equiv j^k + \langle j^k_{\text{связ}} \rangle$$

$$\frac{\partial}{\partial x^k} \langle j^k_{\text{связ}} \rangle = 0, \quad \frac{\partial j^k}{\partial x^k} = 0$$

$$\langle j^k_{\text{связ}} \rangle = c \frac{\partial m^{ki}}{\partial x^i} \quad \frac{\partial^2 m^{ki}}{\partial x^k \partial x^i} = - \frac{\partial^2 m^{ik}}{\partial x^k \partial x^i}$$

$$\langle \rho_{связ} \rangle = \frac{1}{c} \langle j^{0}_{связ} \rangle$$

$$-\operatorname{div} \vec{P} = \frac{\partial m^{0i}}{\partial x^{i}} = \frac{\partial m^{00}}{\partial x^{0}} + \frac{\partial m^{01}}{\partial x^{1}} + \frac{\partial m^{02}}{\partial x^{2}} + \frac{\partial m^{03}}{\partial x^{3}}$$

$$m^{0\alpha} = -(\vec{P})^{\alpha}$$

$$\langle j^{\alpha}_{связ} \rangle = c\,\frac{\partial m^{\alpha i}}{\partial x^{i}} = \left(\frac{\partial \vec{P}}{\partial t}\right)^{\alpha} + c\,(\operatorname{rot} \vec{M})^{\alpha},$$

$$m^{ik} = \begin{pmatrix} 0 & -P_x & -P_y & -P_z \\ P_x & 0 & M_z & -M_y \\ P_y & -M_z & 0 & M_x \\ P_z & M_y & -M_x & 0 \end{pmatrix}$$

$$Q^{ik} = F^{ik} + 4\pi m^{ik}, \quad \frac{\partial Q^{ik}}{\partial x^k} = -\frac{4\pi}{c} j^i.$$

$$Q^{ik} = \begin{pmatrix} 0 & -D_x & -D_y & -D_z \\ +D_x & 0 & -H_z & H_y \\ +D_y & H_z & 0 & -H_x \\ +D_z & -H_y & H_x & 0 \end{pmatrix}.$$

$$\frac{\partial}{\partial x^k} Q^{ik} = -\frac{4\pi}{c} j^i,$$

$$\frac{\partial F_{ik}}{\partial x^m} + \frac{\partial F_{km}}{\partial x^i} + \frac{\partial F_{mi}}{\partial x^k} = 0$$