LECTURE 6

REPEATED GAMES

Introduction

- Lectures 1-5: One-shot games
  - The game is played just once, then the interaction ends.
  - Players have a short term horizon, they are opportunistic, and are unlikely to cooperate (e.g. prisoner's dilemma).
- Firms, individuals, governments often interact over long periods of time
  - Oligopoly
  - Trade partners

Introduction

- Players may behave differently when a game is repeated. They are less opportunistic and prioritize the long-run payoffs, sometimes at the expense of short-term payoffs.

- Types of repeated games:
  - **Finitely repeated**: the game is played for a finite and known number of rounds, e.g. 2 rounds/repetitions.
  - **Infinitely**: the game is repeated infinitely.
  - **Indefinitely repeated**: the game is repeated for an unknown number of times. The interaction will eventually end, but players don't know when.

A model of price competition

- Two firms compete in prices. The NE is to set low prices to gain market shares.
- They could obtain a higher payoff by cooperating (Prisoner's dilemma situation)

Firm 2

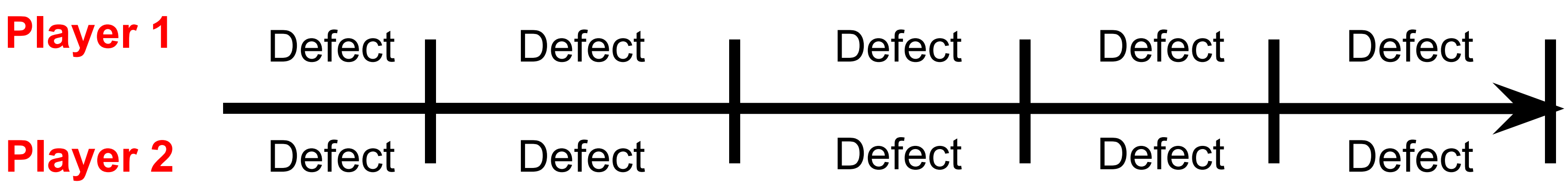
Firm 1		Low (Defect)	High (Cooperate)
	Low(Defect)	288,288	360,216
	High (Cooperate)	216,360	324,324

A model of price competition

- The equilibrium that arises from using dominant strategies is worse for every player than cooperation.
- Why does defection occur?
  - No fear of punishment
  - Short term or myopic play
- What if the game is played “repeatedly” for several periods?
  - The incentive to cooperate may outweigh the incentive to defect.

Finite repetition

- Games where players play the same game for a certain finite number of times. The game is played n times, and n is known in advance.
- Nash Equilibrium:
  - Each player will defect in the very last period
  - Since both know that both will defect in the last period, they also defect in the before last period.
  - etc…until they defect in the first period

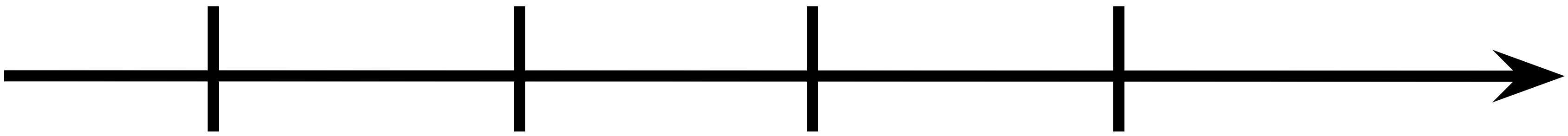


Finite repetition

- When a one-shot game with a unique PSNE is repeated a finite number of times, repetition does not affect the equilibrium outcome. The dominant strategy of defecting will still prevail.
- BUT…finitely repeated games are relatively rare; how often do we really know for certain when a game will end? We routinely play many games that are *indefinitely repeated* (no known end), or *infinitely repeated games*.

Infinite Repetition

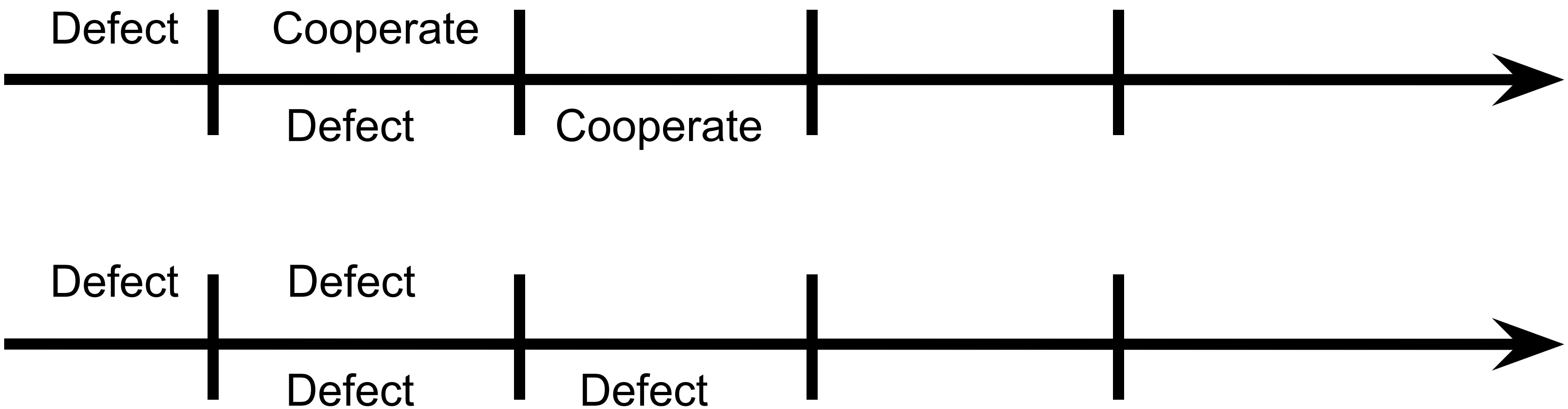
- What if the interaction never ends?



- No final period, so no rollback.
- Players may be using history-dependent strategies, i.e. trigger/contingent strategies:
  - e.g. cooperate as long as the rivals do
  - Upon observing a defection: immediately revert to a period of punishment (i.e. defect) of specified length.

Trigger Strategies

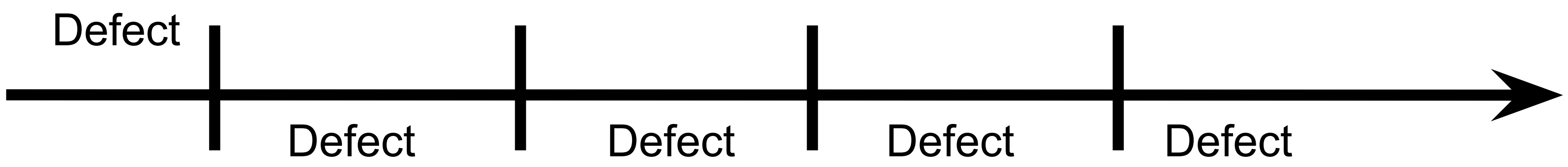
- Tit-for-tat (TFT): choose the action chosen by the other player last period



CONDITIONAL COOPERATION RECIPROCITY

Trigger Strategies

- Grim strategy: cooperate until the other player defects, then if he defects punish him by defecting until the end of the game



Trigger Strategies

- Tit-for-Tat is
  - most forgiving
  - shortest memory
  - proportional
  - credible
    but lacks deterrence

- Grim trigger is
  - least forgiving
  - longest memory
  - not proportional
  - adequate deterrence
    but lacks credibility

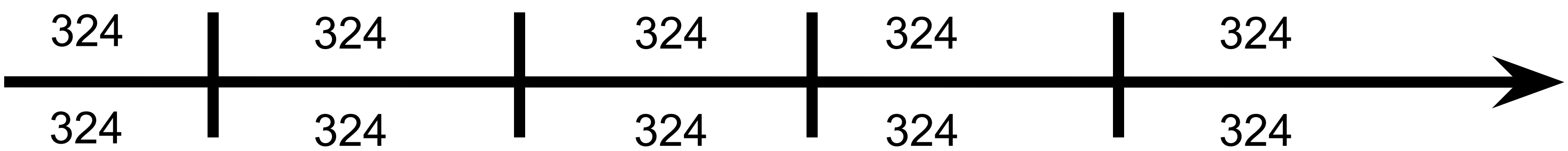
Firm 2

Firm 1

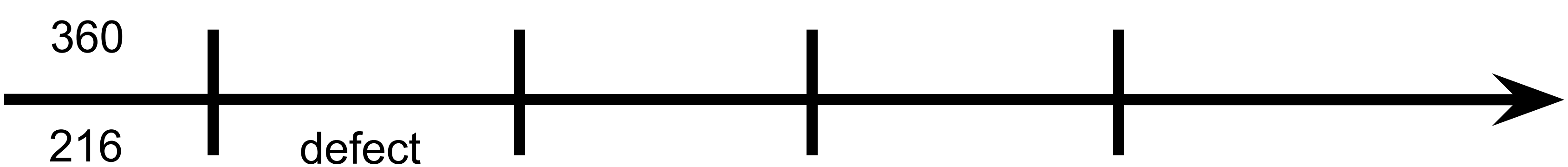
	Low (Defect)	High (Cooperate)
Low (Defect)	288,288	360,216
High (Cooperate)	216,360	324,324

Infinite repetition and defection

- Is it worth defecting? Consider Firm1.
- Cooperation:

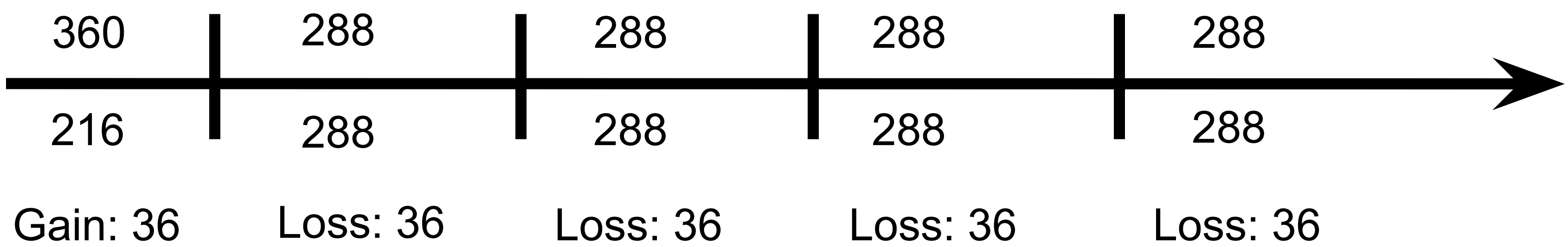


- Firm 1 defects: gain 36 (360-324)
  - If Firm 2 plays TFT, it will also defect next period:

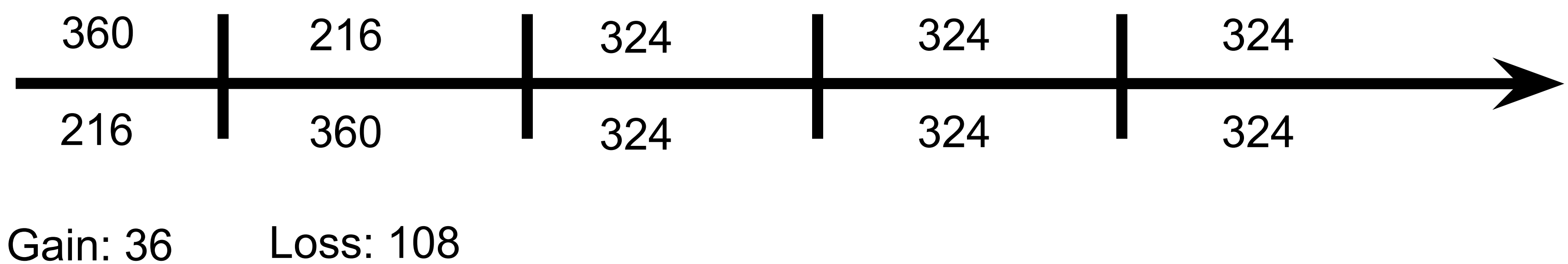


Infinite repetition and defection

- If Firm 1 keeps defecting:



- If Firm 1 reverts back to cooperation:



- If defection, trade-off defection - return to cooperation

Discounting future payoffs

- Recall from the analysis of bargaining that players discount future payoffs. The discount factor is $\delta = 1/(1+r)$, with $\delta < 1$.

- *r* is the interest rate
  - Invest \$1 today → get \$(1+r) next year
  - Want \$1 next year → invest \$1/(1+r) today

- For example, if r=0.25, then $\delta = 0.8$, *i.e.* a player values \$1 received one period in the future as being equivalent to \$0.80 right now.

Discounting future payoffs

- Considering an infinitely repeated game, suppose that an outcome of this game is that a player receives \$1 in every future play (round) of the game, starting from next period.
- Present value of \$1 every period (starting from next period):

$$\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\frac{1}{(1+r)^3}+\frac{1}{(1+r)^4}+\ldots=\frac{1}{r}$$

Defection?

- Defecting once vs. always cooperate against a TFT player. Gain 36 in period 1; Lose 108 in period 2.
  - Defect if: $36 > \frac{108}{1+r} \Rightarrow r > 2$
- Defecting forever vs. always cooperate against a TFT player. Gain 36 in period 1; Lose 36 every period ever after.
  - Defect if: $36 > \frac{36}{r} \Rightarrow r > 1$

Defection?

- When r is high ($r > \text{minimum}\{1,2\}$, i.e. $r > 1$ in this example), cooperation cannot be sustained.
  - When future payoffs are heavily discounted, present gains outweigh future losses.
- Cooperation is sustainable only if $r < 1$, i.e. if future payoffs are not too heavily discounted.

- **Lesson: Infinite repetition increases the possibilities of cooperation, but r has to be low enough.**

Games of unknown length

- Interactions don't last forever: Suppose there is a probability p<1 that the interaction will continue next period Indefinitely repeated games.

- present value of 1 tomorrow is $p\dfrac{1}{1+r}$

- Future losses are discounted more heavily than in infinitely repeated games, because they may not even materialize. Cooperation is more difficult to sustain when p<1 than when p=1.

Games of unknown length

- The effective rate of return R is the rate of return used to discount future payoffs when p<1. R is such that:

$$\frac{1}{1+R} = p\frac{1}{1+r} \Rightarrow R = \frac{1+r}{p} - 1$$

- i.e. the discount factor δ is lower when p<1.
- R>r, and future payoffs are more heavily discounted, which decreases the possibilities of cooperation.

Games of unknown length

- We found that the condition for defecting against a TFT player is:

$$36 > \frac{36}{r} \Rightarrow r > 1$$

- e.g. suppose that r=0.05 □ no defection
- Now assume that there is each period a 10% chance that the game stops: p=0.90.

  □ R=0.16 (still <1, hence no defection) $\frac{1.05}{0.9} - 1$
- If instead p=0.5, then R=1.1, and there is defection (1.1>minimum{1,2}).

Example with asymmetric payoffs

Firm 2

Firm 1		Defect	Cooperate
	Defect	288,300	360,216
	Cooperate	216,360	324,324

Example with asymmetric payoffs

- Firm 1: no change
  - Defect once better than cooperate if:

$$36 > \frac{108}{1+r} \Rightarrow r > 2$$

  - Defect forever better than cooperate if:

$$36 > \frac{36}{r} \Rightarrow r > 1$$

Example with asymmetric payoffs

- Firm 2:
  - Defect once better than cooperate if:

$$36 > \frac{108}{1+r} \Rightarrow r > 2$$

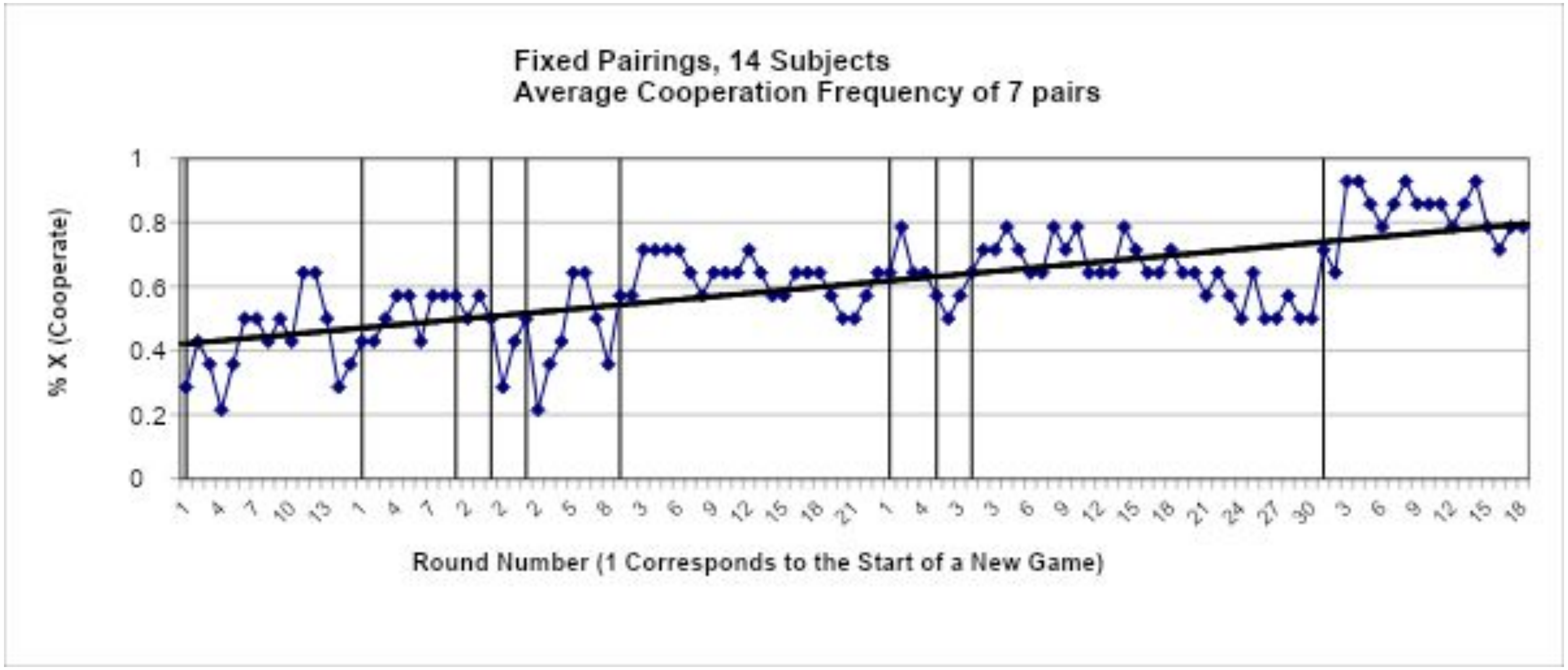
  - Defect forever better than cooperate if:

$$36 > \frac{24}{r} \Rightarrow r > 0.66$$

- Cooperation may not be stable when r>0.66

Experimental evidence from a prisoner's dilemma game

- From Duffy and Ochs (2009), *Games and Economic Behavior.*



- Initially 30% of players cooperate, and this increase to 80% with more repetitions. Trust between players increases over time and fewer of them defect.

The Axelrod Experiment: Assessing trigger strategies

- Axelrod (1980s) invited selected specialists to enter strategies for cooperation games in a round-robin computer tournament.
  - Strategies specified for 200 rounds.
  - TFT obtained the highest overall score in the tournament.
- Why did TFT win?
  - TFT's can adapt to opponents. It resists exploitation by defecting strategies but reciprocates cooperation.
  - Programs that defect suffer against TFT programs.
  - Programs that never defect lost against programs that defect.

The Axelrod Experiment: Assessing trigger strategies

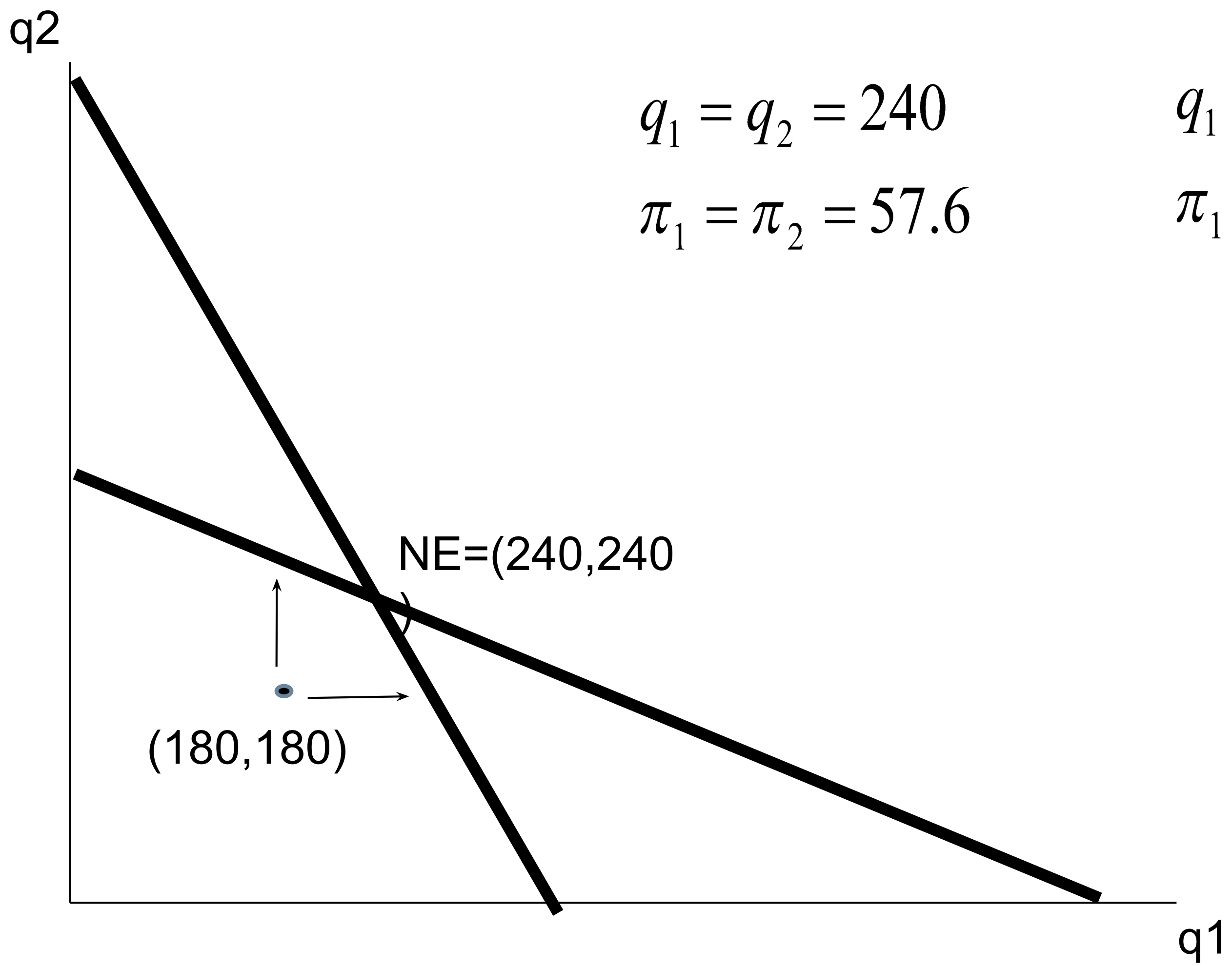
- In another experiment, some "players" were programmed to defect, some to cooperate, some to play trigger strategies such as TFT and grim.
  - The programs that do well "reproduce" themselves and gain in population. The losing programs lose population.
  - After 1000 rounds, TFT accounted for 70% of the population.
  - TFT does well against itself and other cooperative strategies.
  - Defecting strategies fare badly when their own kind spreads, and against TFT.

The Axelrod Experiment: Assessing trigger strategies

- According to Axelrod, TFT follow the following rules:
  - "Don't be envious, don't be the first to defect, reciprocate both cooperation and defection, don't be too clever."

- Folk theorem: two TFT strategies are best replies for each other (i.e. it is a Nash Equilibrium).
- However, other Nash equilibria also exist, and may involve defecting strategies.

Cournot in repeated games



$$q_1 = q_2 = 240$$

$$\pi_1 = \pi_2 = 57.6$$

$$q_1 = q_2 = 180$$

$$\pi_1 = \pi_2 = 64.8$$

Cournot in repeated games

- In a one-shot Cournot game, the unique NE is that producers defect rather than cooperate. Cooperation yields higher payoff, but is not stable.
- Cartels do form, and governments may have to intervene to prevent cartel formation. Some cartels are unstable, but some are stable.

Cournot in repeated games

- How to reconcile the Cournot model with the fact that many cartels are formed?
- Repetition increases the possibilities of cooperation, provided that producers attach sufficient weight on future payoffs (low r).
- “Short-termism” makes cartels less stable.

Cournot in repeated games

- High p also helps.
- Cartels are more likely to be stable in “static” industries, where producers know that they will have a very long-term relationship.
  - e.g. OPEC. The list of oil exporting countries is unlikely to change much over the next decades.
- In “dynamic” industries, where market shares quickly change, collusion is less stable.

Other factors affecting the possibilities of collusion I

- The more complex the negotiations, the greater the costs of cooperation (and create a cartel)
- It is easier to form a cartel when…
  - Few producers are involved.
    - 77% of cartels have six or fewer firms (Connor, 2003)
  - The market is highly concentrated.
    - Cartel members usually control 90%+ of the industry sales (Connor, 2003)
  - Producers have a nearly identical product.
    - If the products are different it is difficult to spot cheating because different products naturally have different prices

Other factors affecting the possibilities of collusion II

- The incentive to defect from the cartel are larger when there are many producers. Consider an industry with *N* producers. π is the monopoly profit.
  - Profit if all producers cooperate: π /N
  - Profit if one defects: become a monopolist and get π
  - Profit if is being punished: 0
- As the number of producers rises, the gain from defection increases:
  - π - π /N increases with N. With a high number of producers, the incentives to defect are strong.

Summary

- One-shot games: defection in equilibrium.
- Having a finite number of repetitions does not increase the possibilities of defection.
- Infinite repetitions can induce players to cooperate, but r has to be low enough.
- Players may use trigger strategies, and experiments suggest that TFT is a strong strategy.
- In indefinitely repeated games, a low p is associated with reduced possibilities of cooperation.