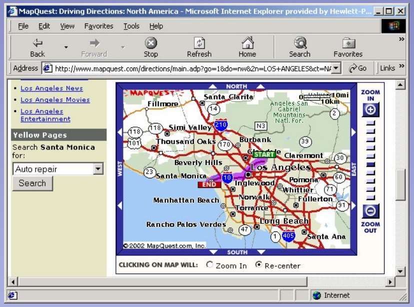
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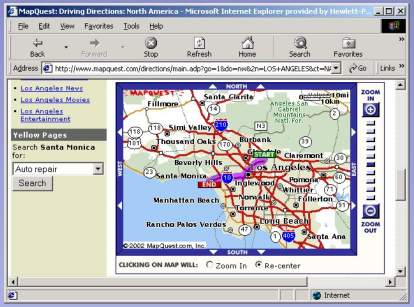
Introduction to Artificial Intelligence
A* Search

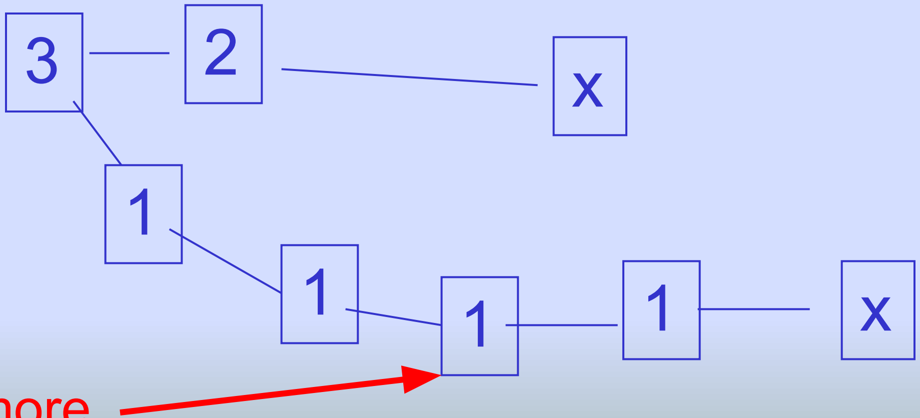
Ruth Bergman

Fall 2004

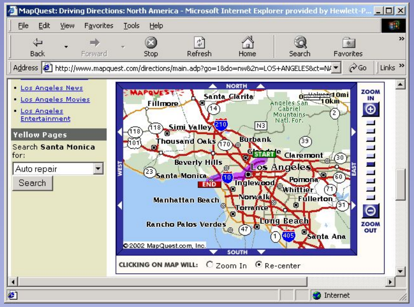


Best-First Search Review

- Advantages
  - Takes advantage of domain information to guide search
  - Greedy advance to the goal
- Disadvantages
  - Considers cost to the goal from the current state
  - Some path can continue to look good according to the heuristic function



At this point the path is more costly than the alternate path

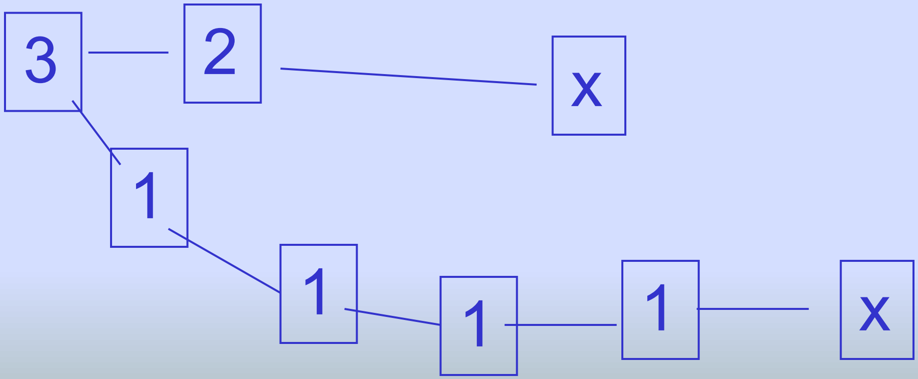


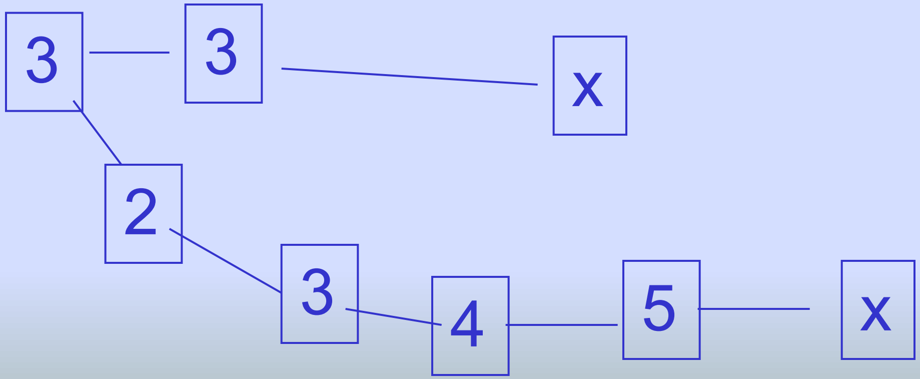
The A* Algorithm

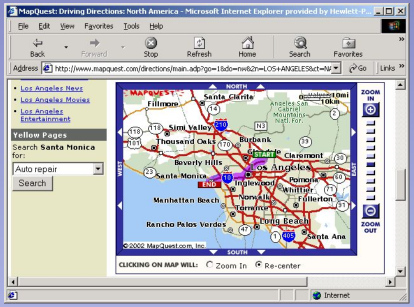
- Consider the overall cost of the solution.

  f(n) = g(n) + h(n)   where g(n) is the path cost to node n

  think of f(n) as an estimate of the cost of the best solution *going through the node n*

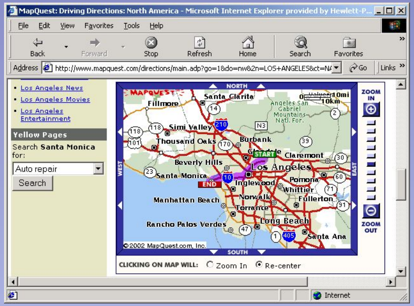






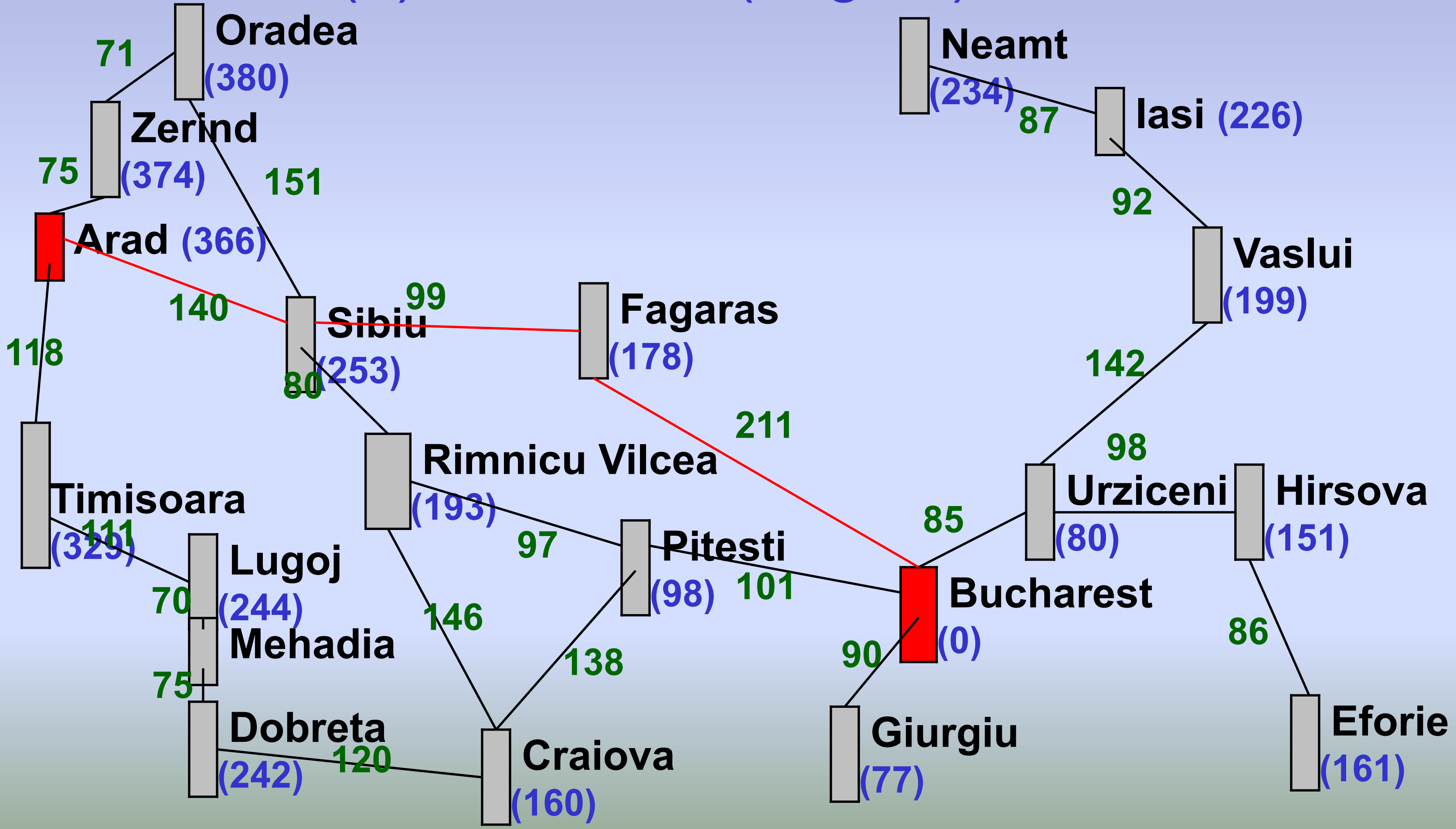
The A* Algorithm

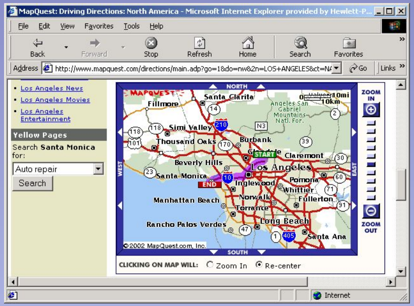
```
A*-Search(initial-test)                    ;; functions cost, h, succ, and GoalTest are defined
   open <- MakePriorityQueue(initial-state, NIL, 0, h(initial-state), h(initial-state))
                        ;; (state, parent, g, h, f)
    while (not(empty(open)))
         node □ pop(open), state □ node-state(node)
         closed □ push (closed, node)
         if GoalTest(state) succeeds return node
         for each child in succ(state)
              new-cost □ node-g(node) + cost(state,child)
              if child in open
                if new-cost < g value of child
                  update(open, child, node, new-cost, h(child), new-cost+h(child))
             elseif child in closed
                if new-cost < g value of child
                  insert(open, child, node, new-cost, h(child), new-cost+h(child))
                  delete(closed,child)
              else
                 open □ push(child, node, new-cost, h(child), new-cost+h(child))
    return failure
```



A* Search: Example

- Travel: h(n) = distance(n, goal)





A* Search : Example

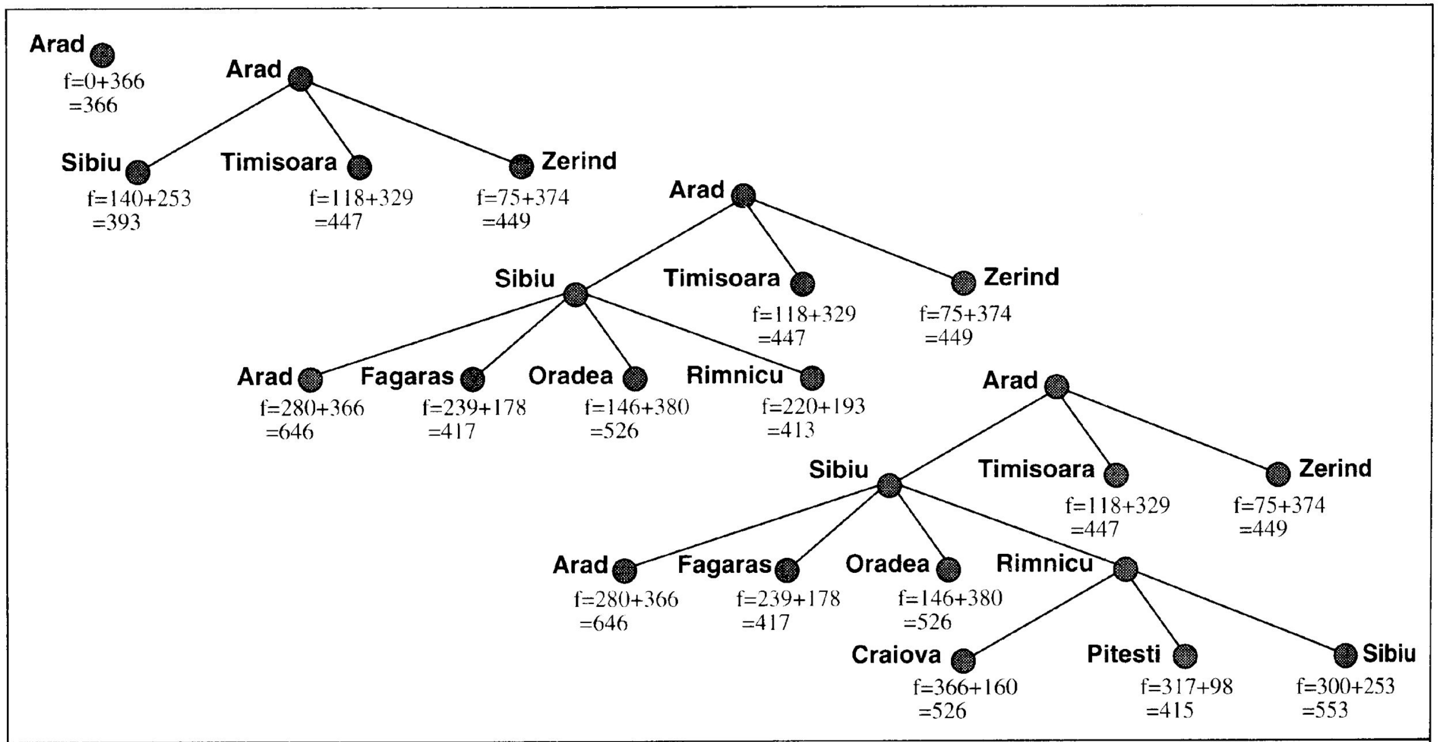
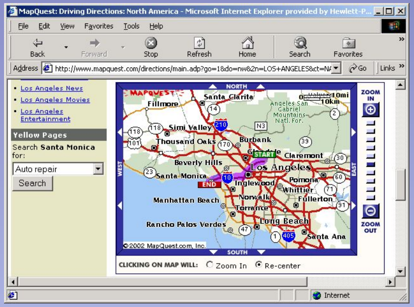
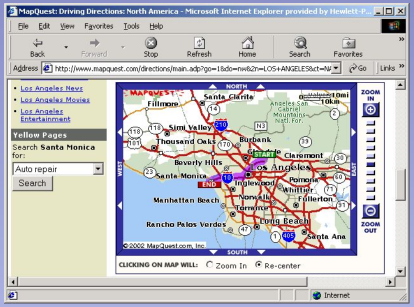


Figure 4.4 Stages in an A* search for Bucharest. Nodes are labelled with $f = g + h$. The h values are the straight-line distances to Bucharest taken from Figure 4.1.



Admissible Heuristics

- we also require h be *admissible:*
  - a heuristic h is admissible if h(n) < h*(n) for all nodes n,
  - where h* is the actual cost of the optimal path from n to the goal
- Examples:
  - travel distance straight line distance must be shorter than actual travel path
  - tiles out of place each move can reorder at most one tile distance of each out of place tile from the correct place each move moves a tile at most one place toward correct place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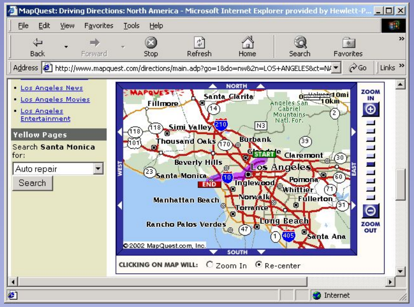
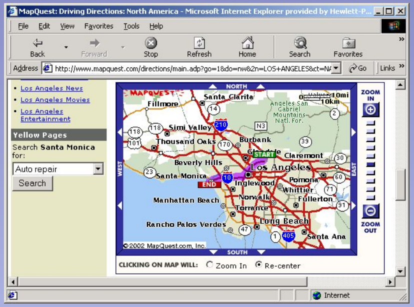
Optimality of A*

- Let us assume that f is non-decreasing along each path
  - if not, simply use parent's value
  - if that's the case, we can think of A* as expanding f contours toward the goal; better heuristics make this contour more "eccentric"
- Let G be an optimal goal state with path cost f*
- Let G_2 be a suboptimal goal state with path cost $g(G_2) > f^*$.
  - suppose A* picks G_2 before G (A* is *not* optimal)
  - suppose n is a leaf node on the path to G when G_2 is chosen
  - if h is admissible, then f* >= f(n)
  - since n was not chosen, it must be the case that $f(n) >= f(G_2)$
  - therefore $f^* >= f(G_2)$, but since G_2 is a goal, $h(G_2)=0$, so $f^* >= g(G_2)$
  - But this is a contradiction --- G_2 is a better goal node than G
  - Thus, our supposition is false and A* is optimal.



Completeness of A*

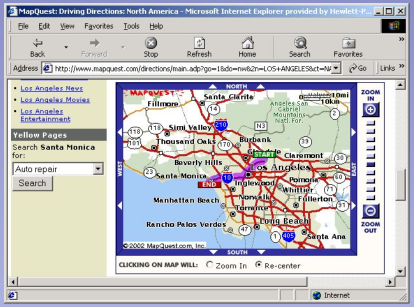
- Suppose there is a goal state G with path cost f*
  - Intuitively: since A* expands nodes in order of increasing f, it must eventually expand node G
- If A* stops and fails
  - Prove by contradiction that this is impossible.
  - There exists a path from the initial state to the node state
  - Let n be the last node expanded along the solution path
  - n has at least one child, that child should be in the open nodes
  - A* does not stop until there are open list is empty (unless it finds a goal state). Contradiction.
- A* is on an infinite path
  - Recall that cost(s1,s2) > δ
  - Let n be the last node expanded along the solution path
  - After f(n)/δ the cumulative cost of the path becomes large enough that A* will expand n. Contradiction.



UCS, BFS, Best-First, and A*

- $f = g + h$ => A* Search
- $h = 0$ => Uniform cost search
- $g = 1$, $h = 0$ => Breadth-First search
- $g = 0$ => Best-First search

Road Map Problem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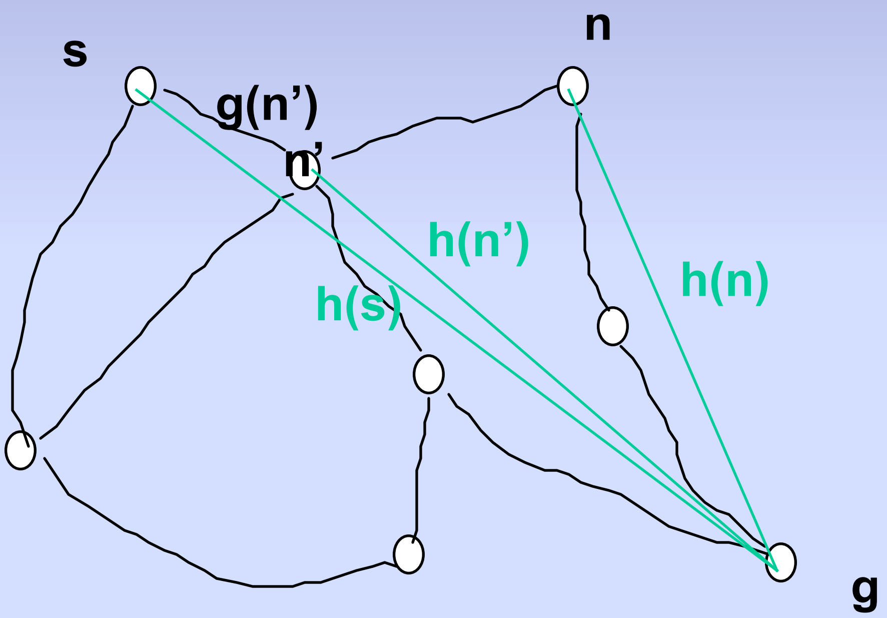

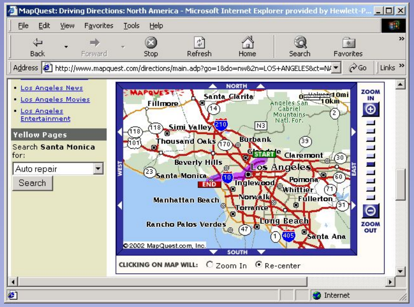
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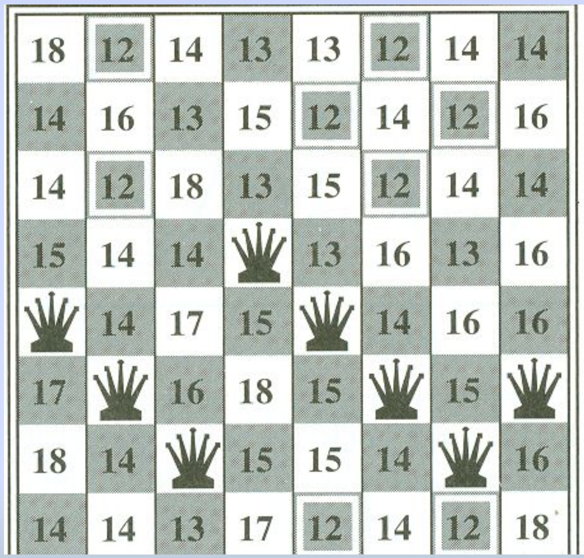
8-queens

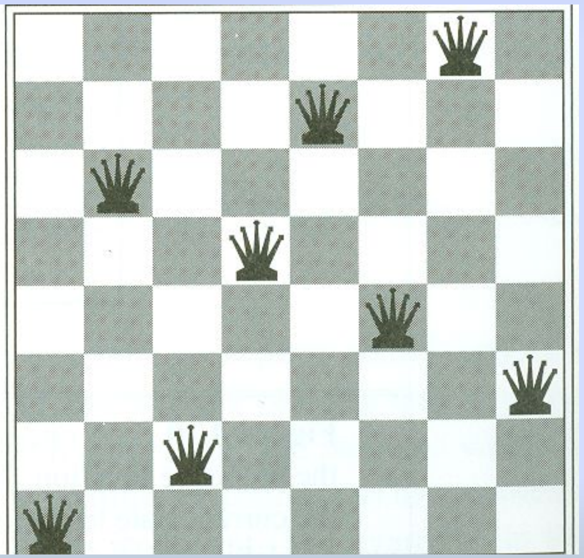
State contains 8 queens on the board

Successor function returns all states generated by moving a single queen to another square in the same column (8*7 = 56 next states)

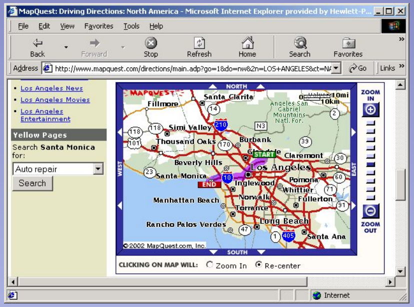
h(s) = number of queens that attack each other in state s.



H(s) = 17



H(s) = 1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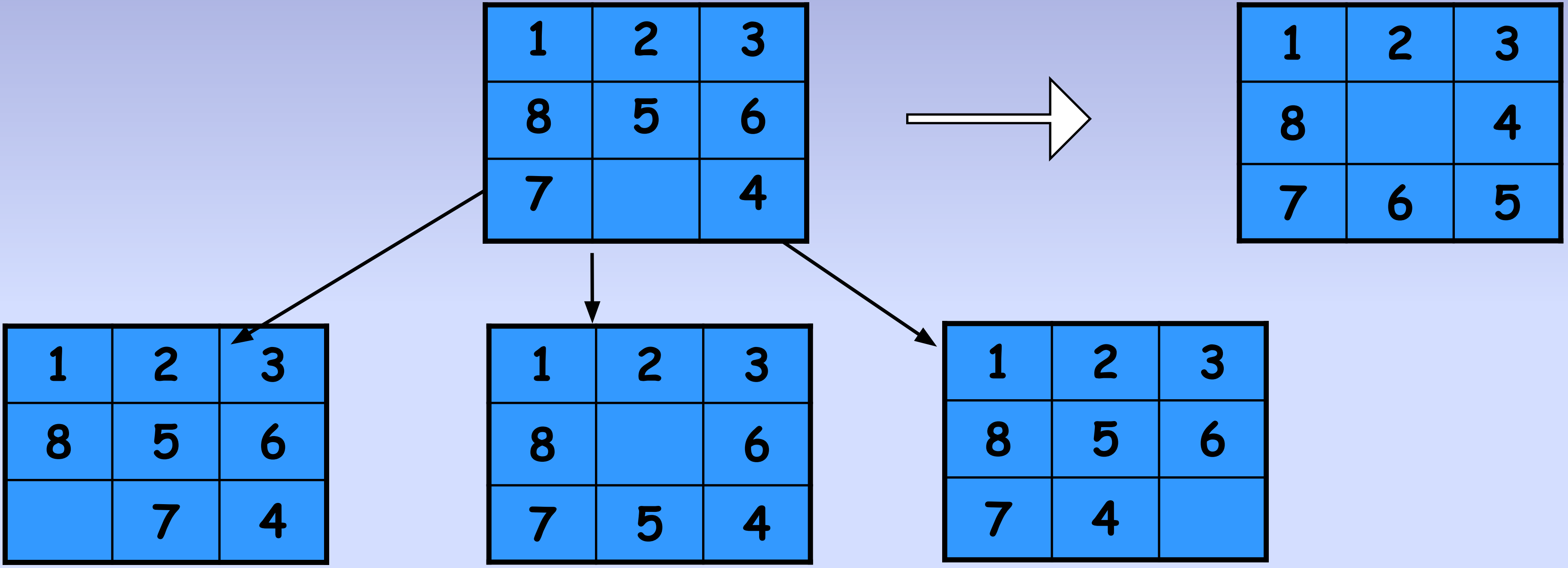
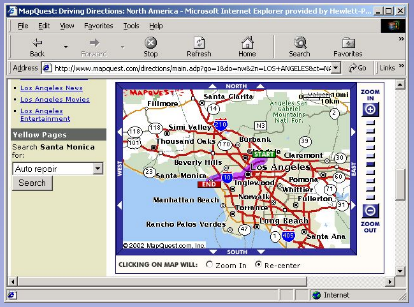
Heuristics : 8 Puzzle





8 Puzzle

- Reachable state : 9!/2 = 181,440

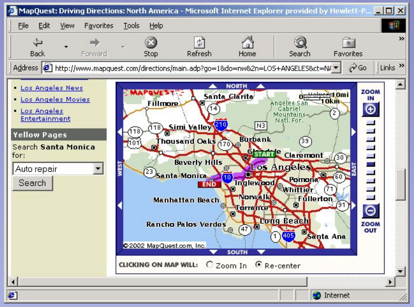
- Use of heuristics
  - h1 : # of tiles that are in the wrong position
  - h2 : sum of Manhattan distance

1	2	3
8	5	6
7		4

h1 = 3

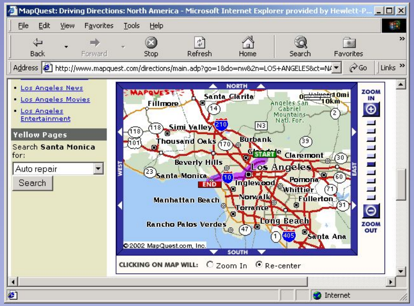
h2 = 1+2+2=5

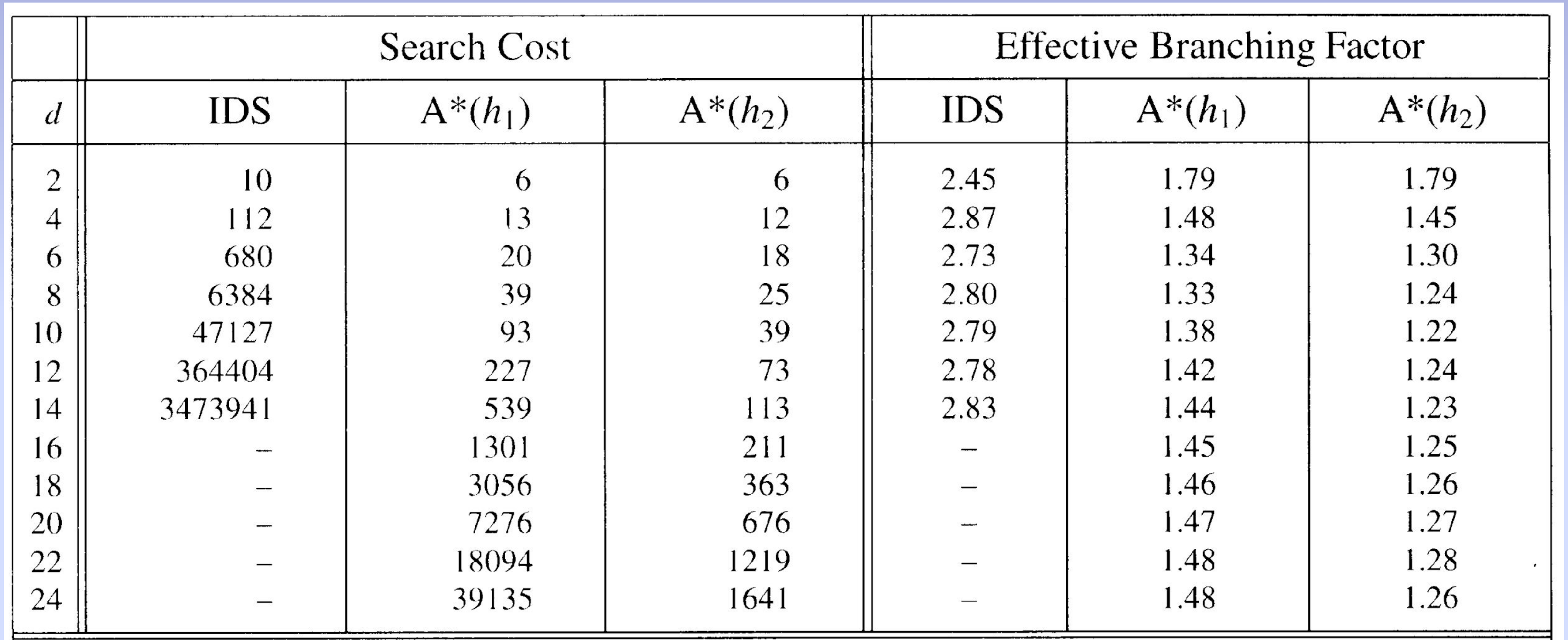
1	2	3
8		4
7	6	5



Effect of Heuristic Accuracy on Performance

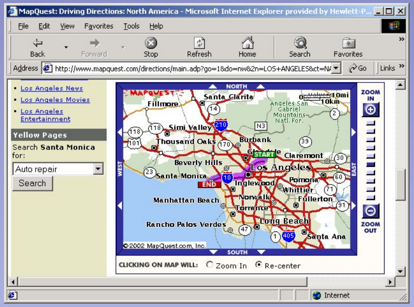
- Well-designed heuristic have its branch close to 1
- h_2 dominates h_1 iff
  $h_2(n) \geq h_1(n), \forall\ n$
- It is always better to use a heuristic function with higher values, as long as it does not overestimate
- Inventing heuristic functions
  - Cost of an exact solution to a relaxed problem is a good heuristic for the original problem
  - collection of admissible heuristics
    $h^*(n) = \max(h_1(n), h_2(n), \ldots, h_k(n))$





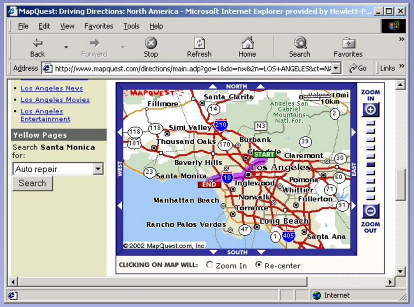
	Search Cost			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 4.8 Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle, for various solution lengths.



A* summary

- Completeness
  - provided finite branching factor and finite cost per operator
- Optimality
  - provided we use an admissible heuristic
- Time complexity
  - worst case is still $O(b^d)$ in some special cases we can do better for a given heuristic
- Space complexity
  - worst case is still $O(b^d)$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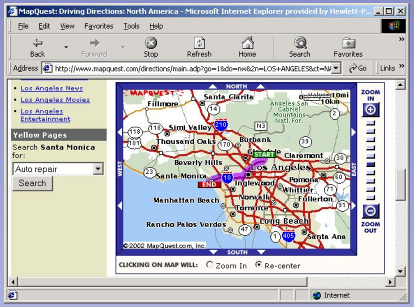
Relax Optimality

- Goals:
  - Minimizing search cost
  - Satisficing solution, i.e. bounded error in the solution

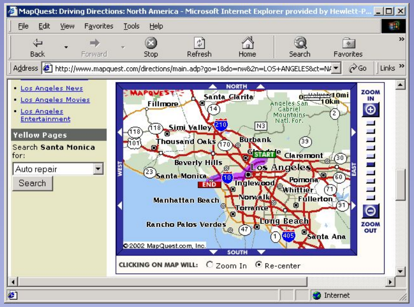
$f(s) = (1-w)\, g(s) + w\, h(s)$

- g can be thought of as the breadth first component
- w = 1 => Best-First search
- w = .5 => A* search
- w = 0 => Uniform search



Iterative Deepening A*

- Goals
  - A storage efficient algorithm that we can use in practice
  - Still complete and optimal
- Modification of A*
  - use f-cost limit as depth bound
  - increase threshold as minimum of f(.) of previous cycle
- Each iteration expands all nodes inside the contour for current f-cost
- same order of node expansion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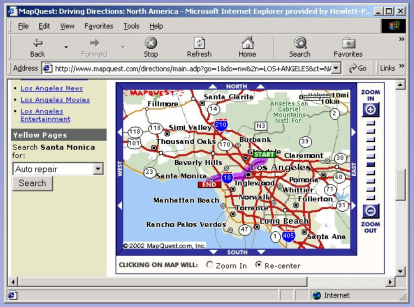
IDA* Algorithm

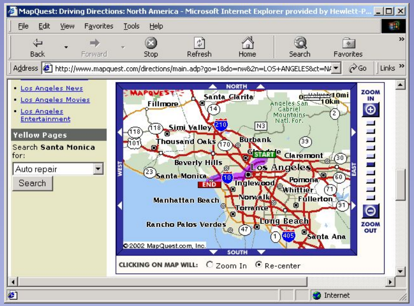
```
IDA* (state,h) returns solution
    f-limit <- h(state)
    loop do
      solution, f-limit □ DFS-Contour(state, f-limit)
      if solution is non-null return solution
      if f-limit = ∞ return failure
    end

DFS-Contour (node,f-limit) returns solution
    if f (node) > f-limit return null, f(node)
    if GoalTest(node) return node, f-limit
    next-f □ ∞
    for each node s in succ(node) do
      solution, new-f □ DFS-Contour(s, f-limit)
      if solution is non-null return solution, f-limit
      next-f □ Min(next-f, new-f)
    end
    return null, next-f
```



IDA* Properties

- Complete:
  - if shortest path fits into memory
- Optimal:
  - if shortest optimal path fits into memory
- Time Complexity: $O(b^{2d})$
- Space Complexity: $O(bd)$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- MapQuest uses a "double Dijkstra" algorithm for its driving directions, working backward from both the starting and ending points at once. MapQuest uses a "double Dijkstra" algorithm for its driving directions, working backward from both the starting and ending points at once.
- the algorithm uses heuristic tricks to minimize the size of the graph that must be searched.