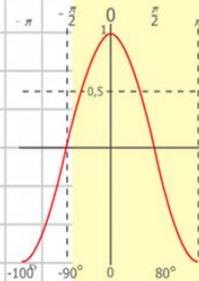
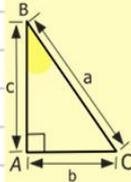
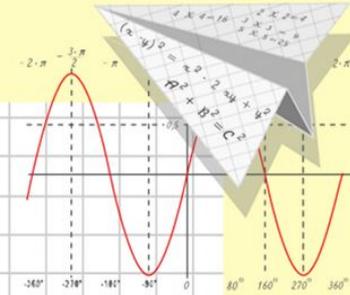
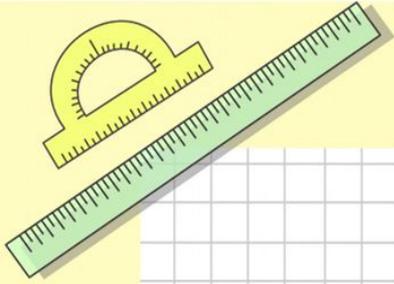


Математик

а

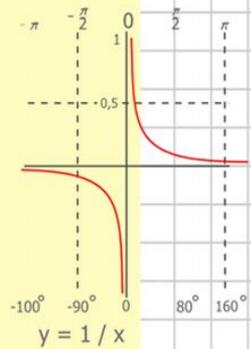
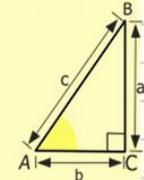
Занятие 110. Определенный интеграл

1. Криволинейная трапеция
2. Определенный интеграл
3. Формула Ньютона-Лейбница



$$y = \cos x$$

$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \end{aligned}$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

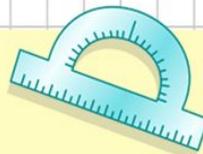


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

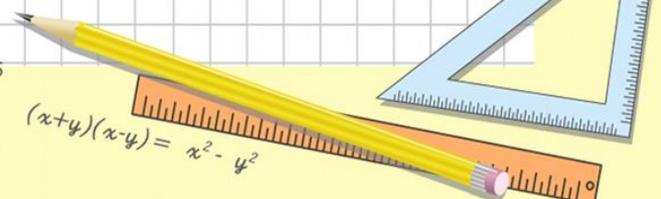


$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

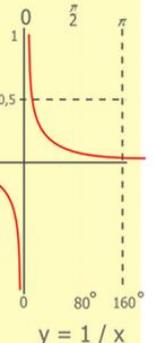
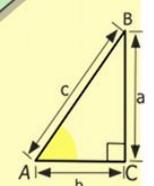
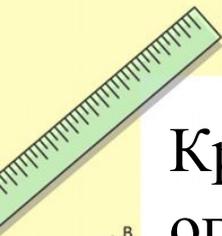
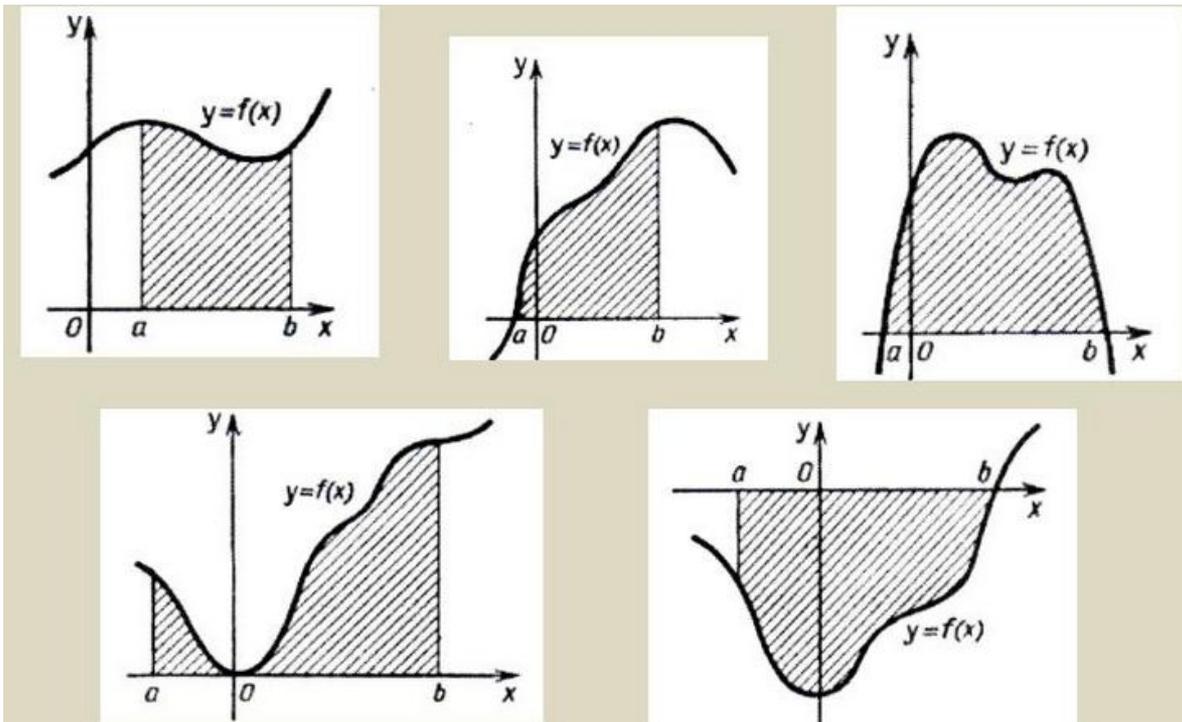
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



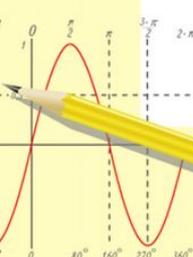
$$(x+y)(x-y) = x^2 - y^2$$

Криволинейная трапеция

Криволинейной трапецией называется фигура, ограниченная осью OX , вертикальными прямыми $x=a$ и $x=b$, а также графиком функции $y=f(x)$, которая на отрезке $[a; b]$ непрерывна и сохраняет постоянный знак.



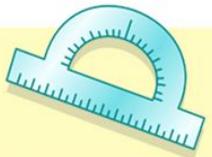
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

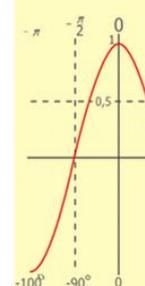
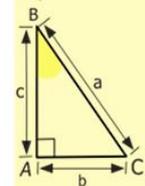
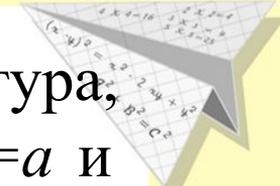
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

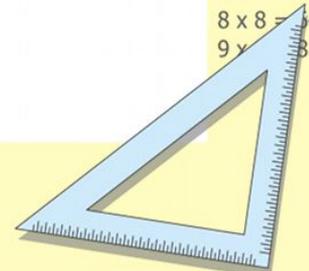
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



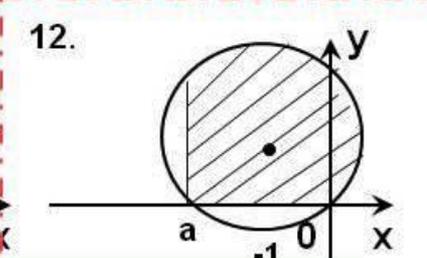
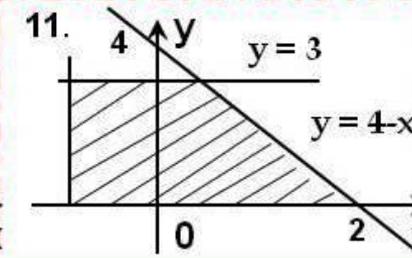
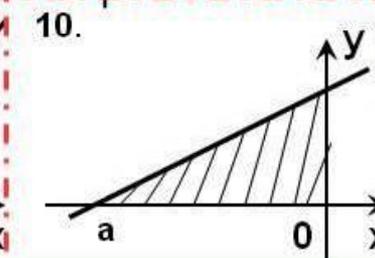
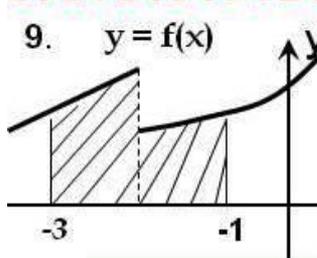
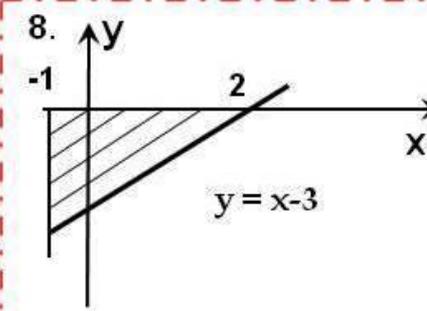
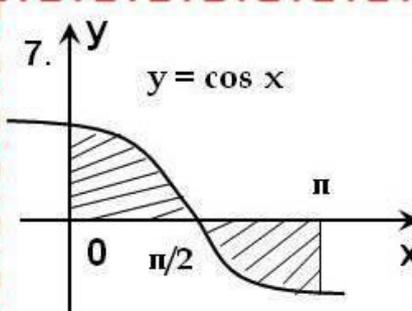
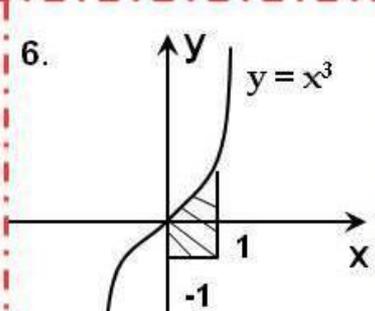
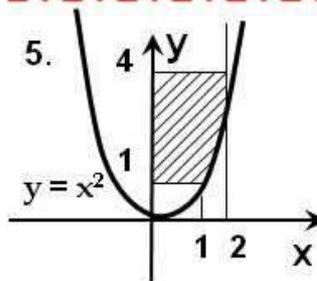
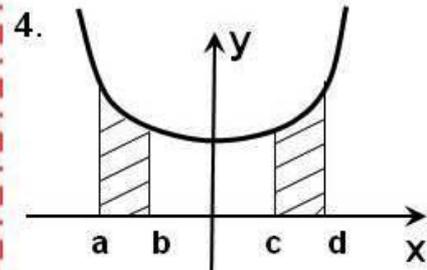
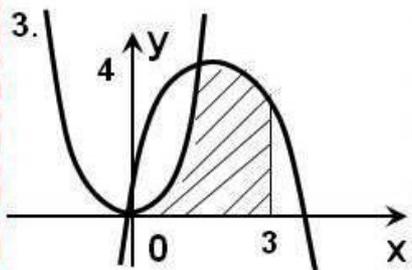
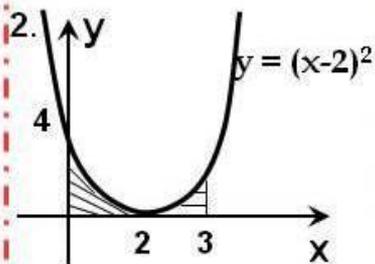
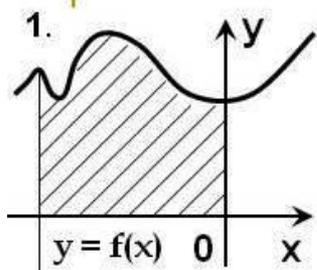
$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Криволинейная трапеция

На каких рисунках изображены криволинейные трапеции?



Ответ: 1,2,8,10

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

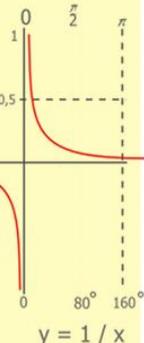
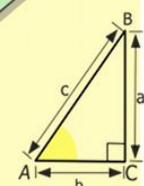
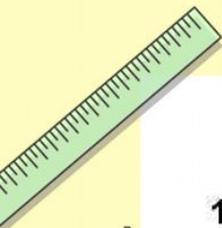
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

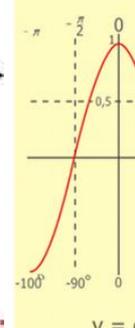
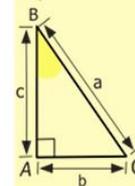
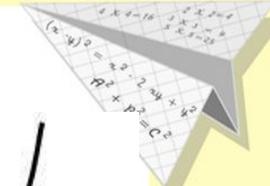
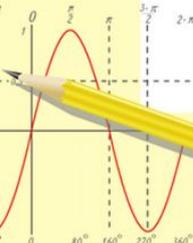
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

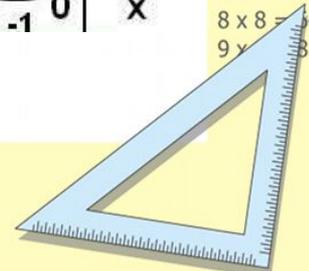
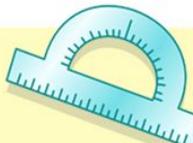
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 21\ 0 \\ + 84\ 0 \\ \hline 105\ 0\ 00 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Определенный интеграл

Определенным интегралом функции $y=f(x)$ в пределах от a до b называют площадь соответствующей криволинейной трапеции

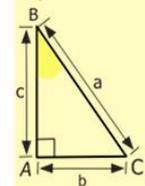
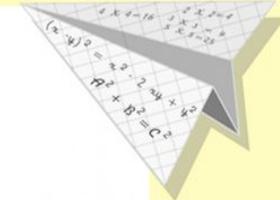
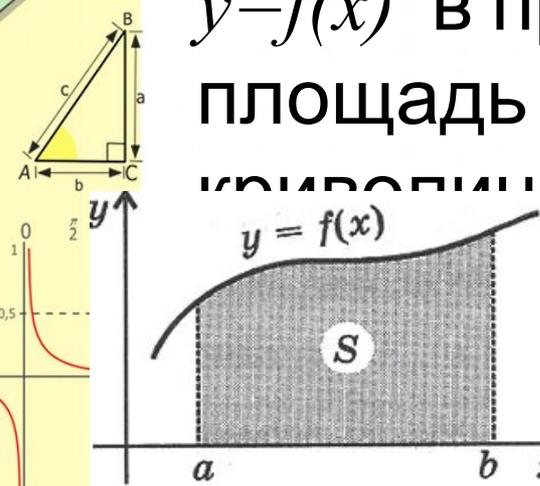
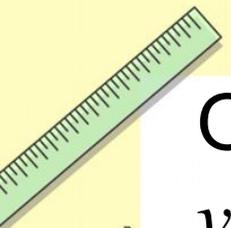
$$\int_a^b f(x) dx = S(\text{кр.трап.})$$

пределы
интегрирования

знак
интеграла

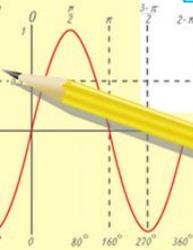
формула
функции

дифференциал
функции



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

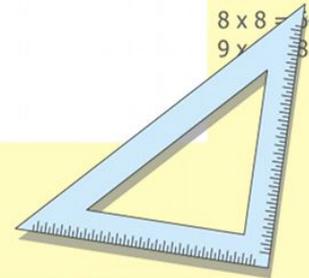
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$
$$\frac{x}{70}$$

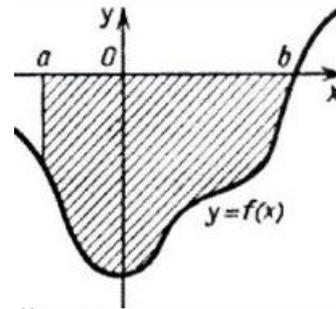
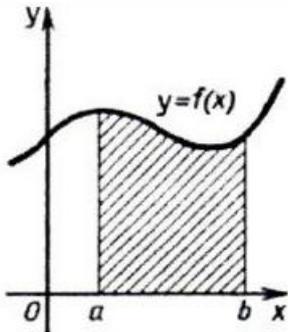
$$(x+y)(x-y) = x^2 - y^2$$



Криволинейная трапеция

Решая вопрос о площади криволинейной трапеции, пришли к выводу, что она будет равна приращению первообразной $F(x)$ от функции $f(x)$, которая ограничивает эту трапецию сверху.

$$S = F(x) \Big|_a^b = F(b) - F(a)$$



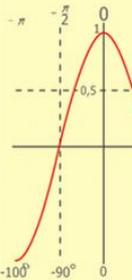
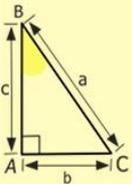
$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

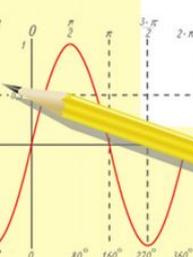
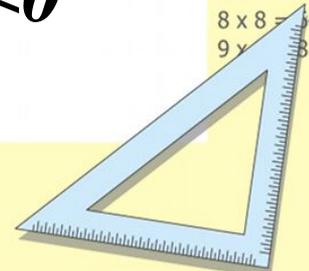
$$\begin{cases} x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$y = \cos$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

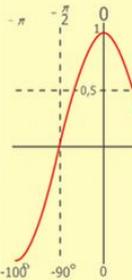
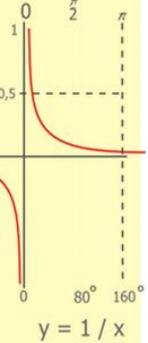
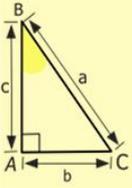
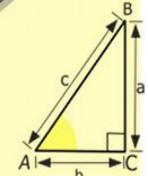
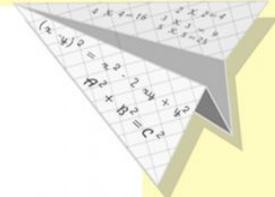
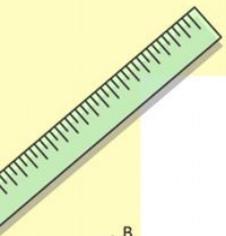
Формула Ньютона-Лейбница

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_1^2 x^4 dx = \frac{x^5}{5} \Big|_1^2 = \frac{2^5}{5} - \frac{1^5}{5} = \frac{32}{5} - \frac{1}{5} = \frac{31}{5} = 6,2$$

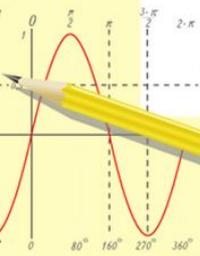
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_4^9 \sqrt{x} dx = \frac{2}{3} \sqrt{x^3} \Big|_4^9 = \frac{2}{3} \sqrt{9^3} - \frac{2}{3} \sqrt{4^3} = \frac{2}{3} * (27 - 8) = \frac{38}{3}$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

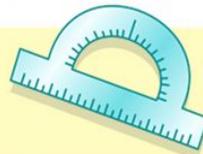
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

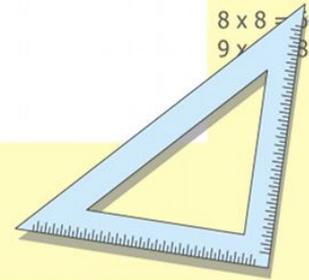


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

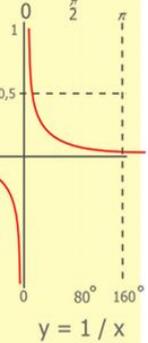
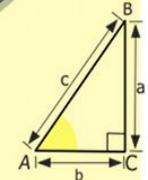
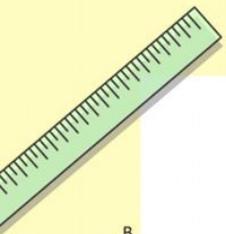


Формула Ньютона-Лейбница

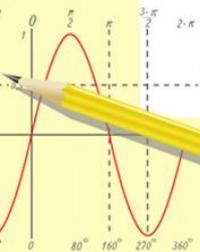
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\begin{aligned} \int_0^2 (6x^2 + 4x - 5) dx &= 6 * \frac{x^3}{3} + 4 * \frac{x^2}{2} - 5x \Big|_0^2 = 2x^3 + 2x^2 - 5x \Big|_0^2 = \\ &= (2 * 2^3 + 2 * 2^2 - 5 * 2) - (2 * 0^3 + 2 * 0^2 - 5 * 0) = \\ &= 16 + 8 - 10 - 0 = 14 \end{aligned}$$

$$\int_0^1 \sqrt[5]{x^2} dx = \int_0^1 x^{\frac{2}{5}} dx = \frac{x^{\frac{7}{5}}}{\frac{7}{5}} \Big|_0^1 = \frac{5}{7} \sqrt[5]{x^7} \Big|_0^1 = \frac{5}{7} * \sqrt[5]{1^7} - \frac{5}{7} * \sqrt[5]{0^7} = \frac{5}{7}$$



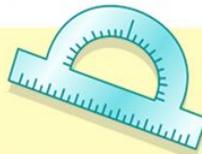
$$\begin{array}{r} 1 \ 2 \ 5 \ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

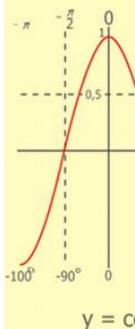
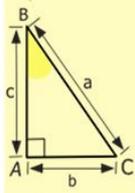


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

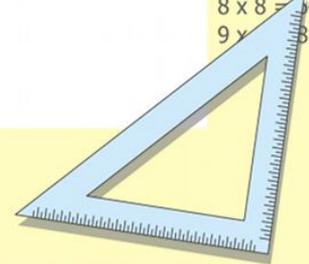
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



- 2 x 2 = 4
- 3 x 3 = 9
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Формула Ньютона-Лейбница

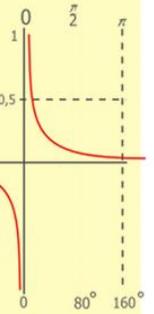
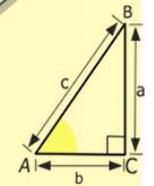
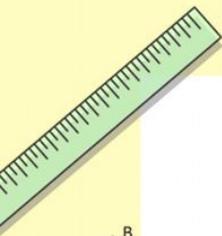
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_1^4 \frac{dx}{5x-2} = \left| \begin{array}{l} kx+b=5x-2, k=5 \\ f(t) = \frac{1}{t} \Rightarrow F(t) = \ln|t| \end{array} \right| = \frac{1}{5} \ln|5x-2| \Big|_1^4 =$$

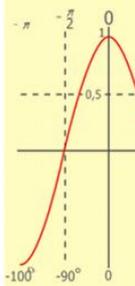
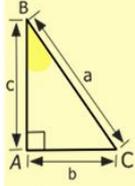
$$= \frac{1}{5} \ln|5 \cdot 4 - 2| - \frac{1}{5} \ln|5 \cdot 1 - 2| = \frac{1}{5} (\ln 18 - \ln 3) = \frac{1}{5} \ln \frac{18}{3} = \frac{1}{5} \ln 6$$

$$\int_{-1}^1 \frac{dx}{(2x+3)^3} = \left| \begin{array}{l} kx+b=2x+3, k=2 \\ f(t) = \frac{1}{t^3} = t^{-3} \Rightarrow F(t) = \frac{t^{-2}}{-2} = -\frac{1}{2t^2} \end{array} \right| =$$

$$= \frac{1}{2} * \left(-\frac{1}{2(2x+3)^2} \right) \Big|_{-1}^1 = -\frac{1}{4(2x+3)^2} \Big|_{-1}^1 = -0,01 - (-0,25) = 0,24$$

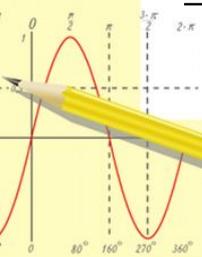


$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105\ 000 \end{array}$



$y = \cos$

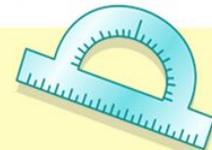
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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

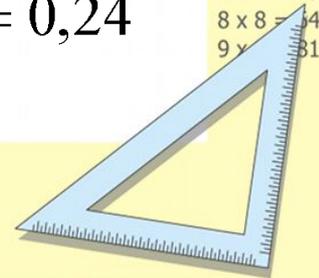


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Формула Ньютона-Лейбница

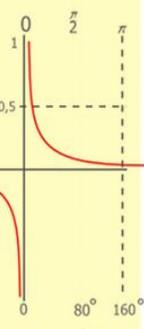
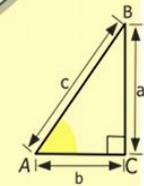
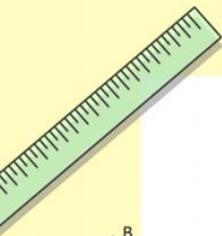
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_0^{\pi/12} \cos 4x dx = \left| \begin{array}{l} kx + b = 4x, k = 4 \\ f(t) = \cos t \Rightarrow F(t) = \sin t \end{array} \right| = \frac{1}{4} * \sin 4x \Big|_0^{\pi/12} =$$

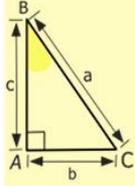
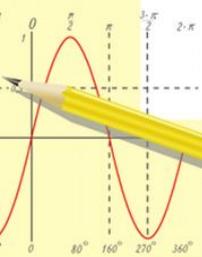
$$= \frac{1}{4} * \sin\left(4 * \frac{\pi}{12}\right) - \frac{1}{4} * \sin(4 * 0) = \frac{1}{4} * \sin \frac{\pi}{3} - 0 = \frac{1}{4} * \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

$$\int_0^1 e^{3x-1} dx = \left| \begin{array}{l} kx + b = 3x - 1, k = 3 \\ f(t) = e^t \Rightarrow F(t) = e^t \end{array} \right| = \frac{1}{3} * e^{3x-1} \Big|_0^1 = \frac{1}{3} * e^2 - \frac{1}{3} * e^{-1} =$$

$$= \frac{e^2}{3} - \frac{1}{3e} = \frac{e^3 - 1}{3e}$$



$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$

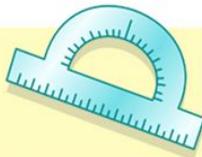


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