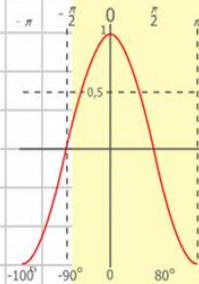
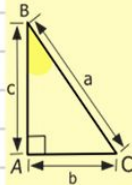
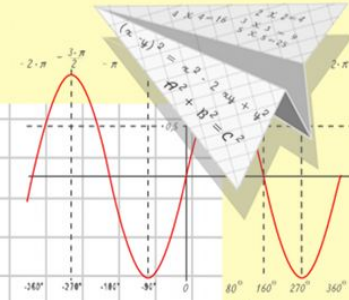
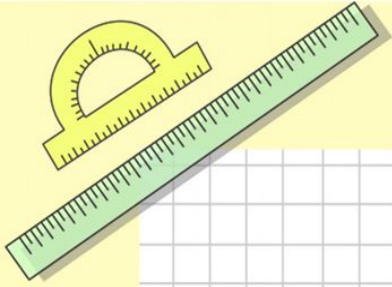


# Математик

а

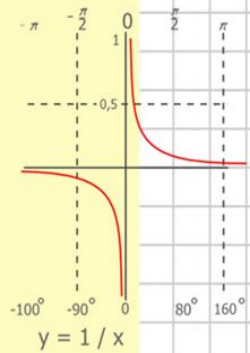
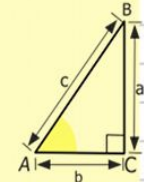
## Занятие 110. Определенный интеграл

1. Криволинейная трапеция
2. Определенный интеграл
3. Формула Ньютона-Лейбница



$$y = \cos x$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



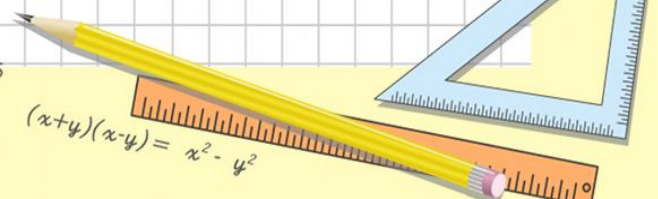
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

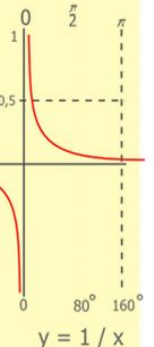
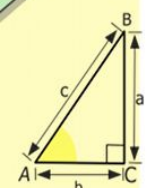
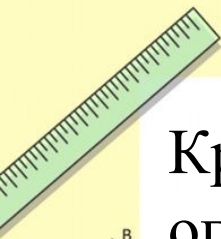
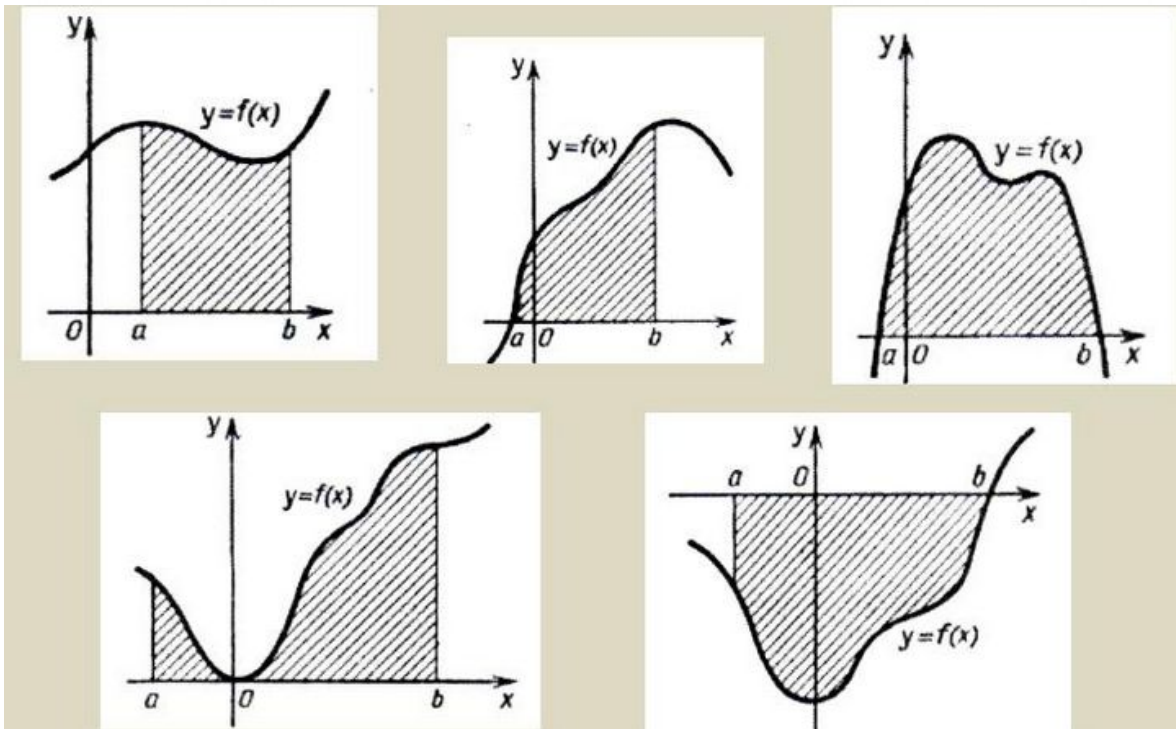
$$x = 70$$



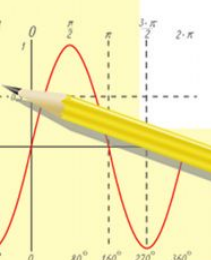
$$(x+y)(x-y) = x^2 - y^2$$

# Криволинейная трапеция

Криволинейной трапецией называется фигура, ограниченная осью  $OX$ , вертикальными прямыми  $x=a$  и  $x=b$ , а также графиком функции  $y=f(x)$ , которая на отрезке  $[a; b]$  непрерывна и сохраняет постоянный знак.



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105\ 000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

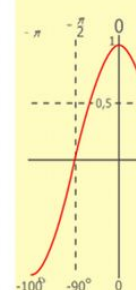
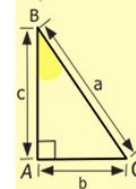
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

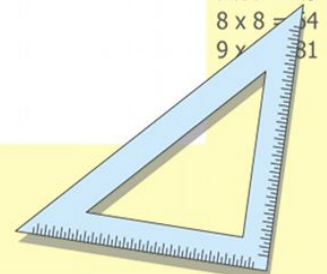
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



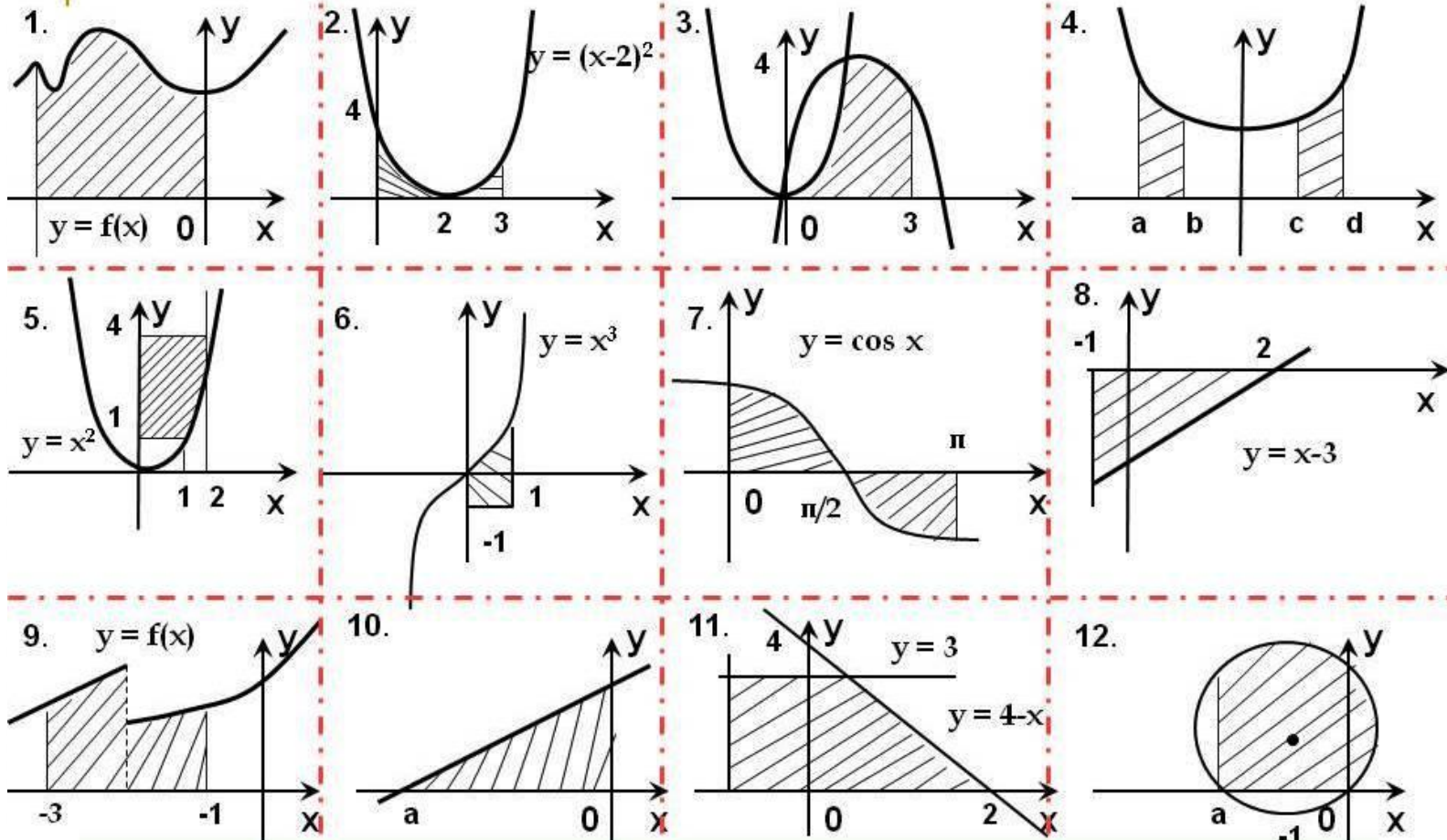
$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Криволинейная трапеция

На каких рисунках изображены криволинейные трапеции?



Ответ: 1,2,8,10

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

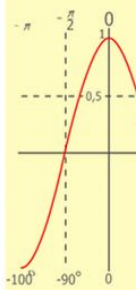
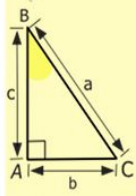
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

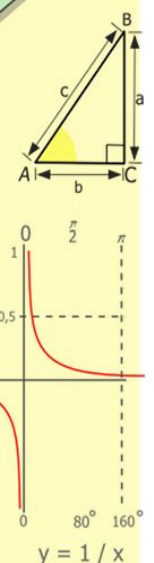
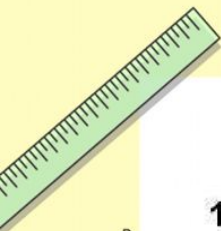
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

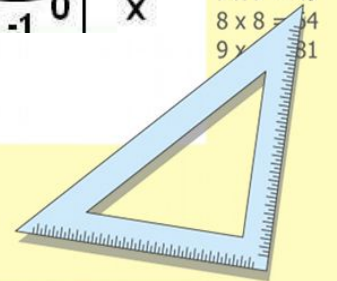
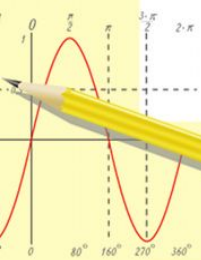


$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 21\ 0 \\ + 84\ 0 \\ \hline 105\ 0\ 00 \end{array}$$





# Определенный интеграл

Определенным интегралом функции  $y=f(x)$  в пределах от  $a$  до  $b$  называют площадь соответствующей криволинейной трапеции

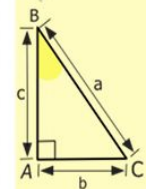
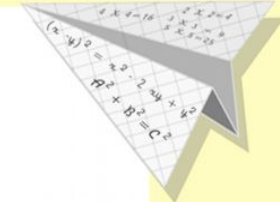
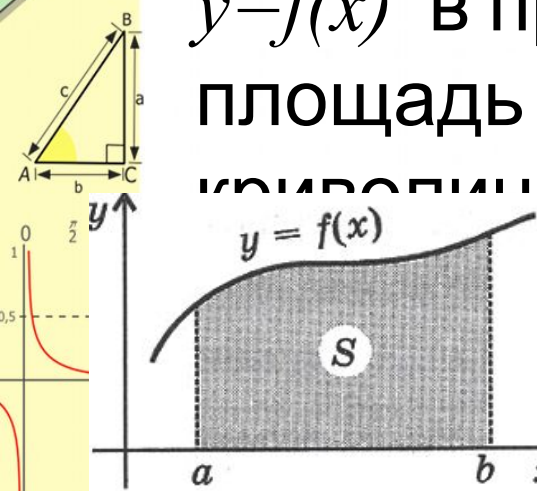
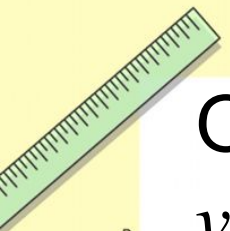
$$\int_a^b f(x) dx = S(\text{кр.трап.})$$

пределы  
интегрирования

знак  
интеграла

формула  
функции

дифференциал  
функции



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

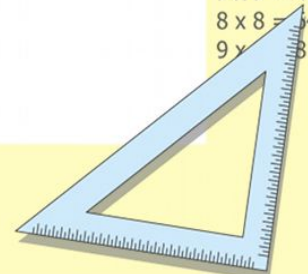
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$
$$\frac{x}{70}$$

$$(x+y)(x-y) = x^2 - y^2$$

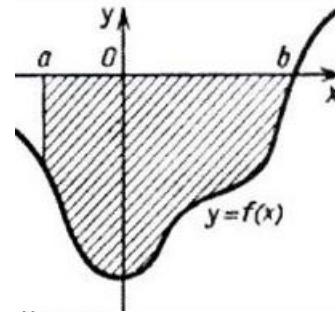
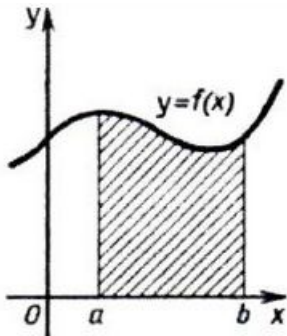
- 2 x 2 = 4
- 3 x 3 = 9
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- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Криволинейная трапеция

Решая вопрос о площади криволинейной трапеции, пришли к выводу, что она будет равна приращению первообразной  $F(x)$  от функции  $f(x)$ , которая ограничивает эту трапецию сверху.

$$S = F(x) \Big|_a^b = F(b) - F(a)$$



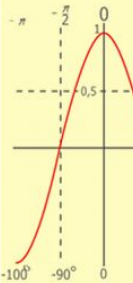
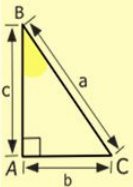
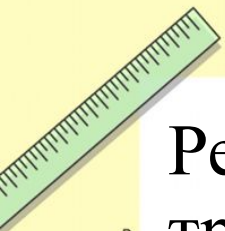
$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

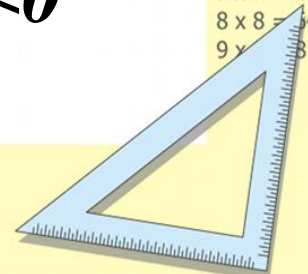
$$\begin{cases} x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$y = \cos$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

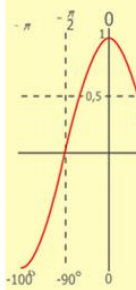
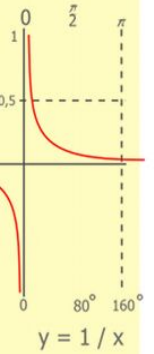
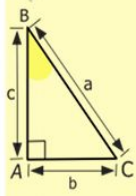
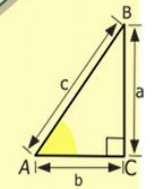
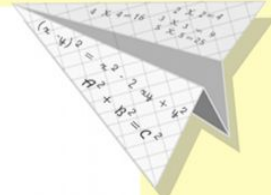
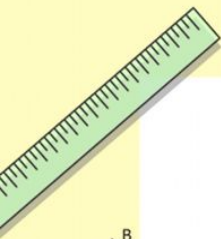
# Формула Ньютона-Лейбница

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_1^2 x^4 dx = \frac{x^5}{5} \Big|_1^2 = \frac{2^5}{5} - \frac{1^5}{5} = \frac{32}{5} - \frac{1}{5} = \frac{31}{5} = 6,2$$

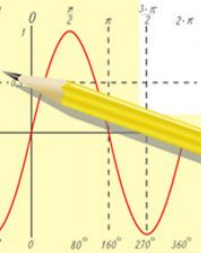
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_4^9 \sqrt{x} dx = \frac{2}{3} \sqrt{x^3} \Big|_4^9 = \frac{2}{3} \sqrt{9^3} - \frac{2}{3} \sqrt{4^3} = \frac{2}{3} * (27 - 8) = \frac{38}{3}$$



$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

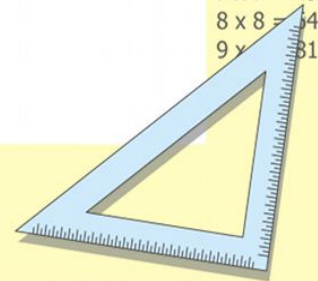
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

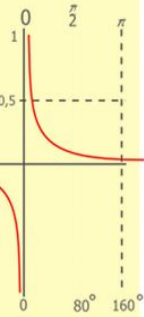
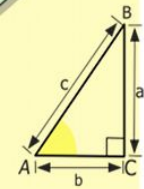
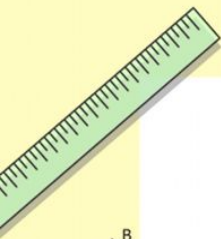


# Формула Ньютона-Лейбница

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

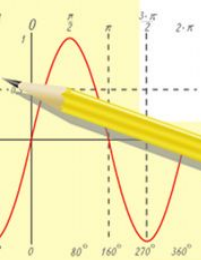
$$\begin{aligned} \int_0^2 (6x^2 + 4x - 5) dx &= 6 * \frac{x^3}{3} + 4 * \frac{x^2}{2} - 5x \Big|_0^2 = 2x^3 + 2x^2 - 5x \Big|_0^2 = \\ &= (2 * 2^3 + 2 * 2^2 - 5 * 2) - (2 * 0^3 + 2 * 0^2 - 5 * 0) = \\ &= 16 + 8 - 10 - 0 = 14 \end{aligned}$$

$$\int_0^1 \sqrt[5]{x^2} dx = \int_0^1 x^{\frac{2}{5}} dx = \frac{x^{\frac{7}{5}}}{\frac{7}{5}} \Big|_0^1 = \frac{5}{7} \sqrt[5]{x^7} \Big|_0^1 = \frac{5}{7} * \sqrt[5]{1^7} - \frac{5}{7} * \sqrt[5]{0^7} = \frac{5}{7}$$



$y = 1/x$

$$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

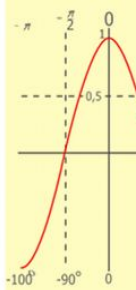
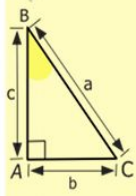
$\sin 90^\circ = 1$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

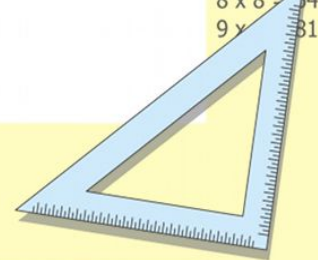
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$y = \cos$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
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# Формула Ньютона-Лейбница

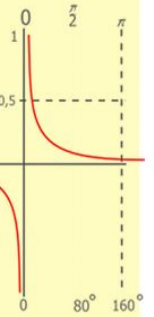
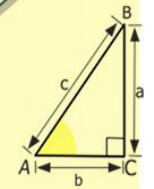
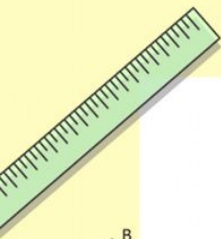
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_1^4 \frac{dx}{5x-2} = \left| \begin{array}{l} kx+b=5x-2, k=5 \\ f(t) = \frac{1}{t} \Rightarrow F(t) = \ln|t| \end{array} \right| = \frac{1}{5} \ln|5x-2| \Big|_1^4 =$$

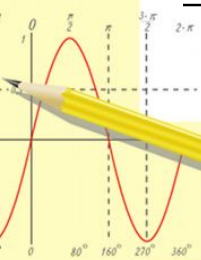
$$= \frac{1}{5} \ln|5 \cdot 4 - 2| - \frac{1}{5} \ln|5 \cdot 1 - 2| = \frac{1}{5} (\ln 18 - \ln 3) = \frac{1}{5} \ln \frac{18}{3} = \frac{1}{5} \ln 6$$

$$\int_{-1}^1 \frac{dx}{(2x+3)^3} = \left| \begin{array}{l} kx+b=2x+3, k=2 \\ f(t) = \frac{1}{t^3} = t^{-3} \Rightarrow F(t) = \frac{t^{-2}}{-2} = -\frac{1}{2t^2} \end{array} \right| =$$

$$= \frac{1}{2} * \left( -\frac{1}{2(2x+3)^2} \right) \Big|_{-1}^1 = -\frac{1}{4(2x+3)^2} \Big|_{-1}^1 = -0,01 - (-0,25) = 0,24$$



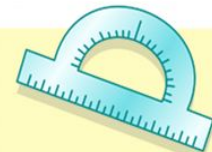
$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105\ 000 \end{array}$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

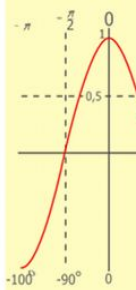
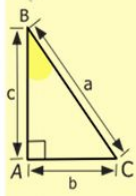
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



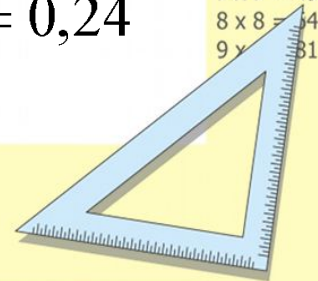
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
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- 8 x 8 = 64
- 9 x 9 = 81





# Формула Ньютона-Лейбница

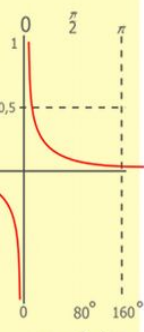
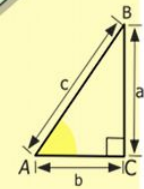
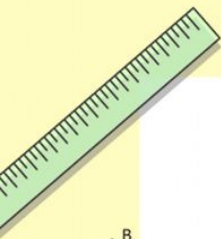
$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

$$\int_0^{\pi/12} \cos 4x dx = \left| \begin{array}{l} kx + b = 4x, k = 4 \\ f(t) = \cos t \Rightarrow F(t) = \sin t \end{array} \right| = \frac{1}{4} * \sin 4x \Big|_0^{\pi/12} =$$

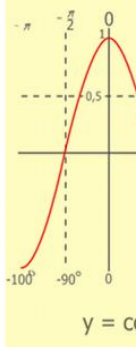
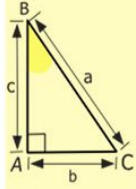
$$= \frac{1}{4} * \sin\left(4 * \frac{\pi}{12}\right) - \frac{1}{4} * \sin(4 * 0) = \frac{1}{4} * \sin \frac{\pi}{3} - 0 = \frac{1}{4} * \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

$$\int_0^1 e^{3x-1} dx = \left| \begin{array}{l} kx + b = 3x - 1, k = 3 \\ f(t) = e^t \Rightarrow F(t) = e^t \end{array} \right| = \frac{1}{3} * e^{3x-1} \Big|_0^1 = \frac{1}{3} * e^2 - \frac{1}{3} * e^{-1} =$$

$$= \frac{e^2}{3} - \frac{1}{3e} = \frac{e^3 - 1}{3e}$$



$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ \times 42 \\ \hline 2100 \\ + 8400 \\ \hline 105000 \end{array}$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

