

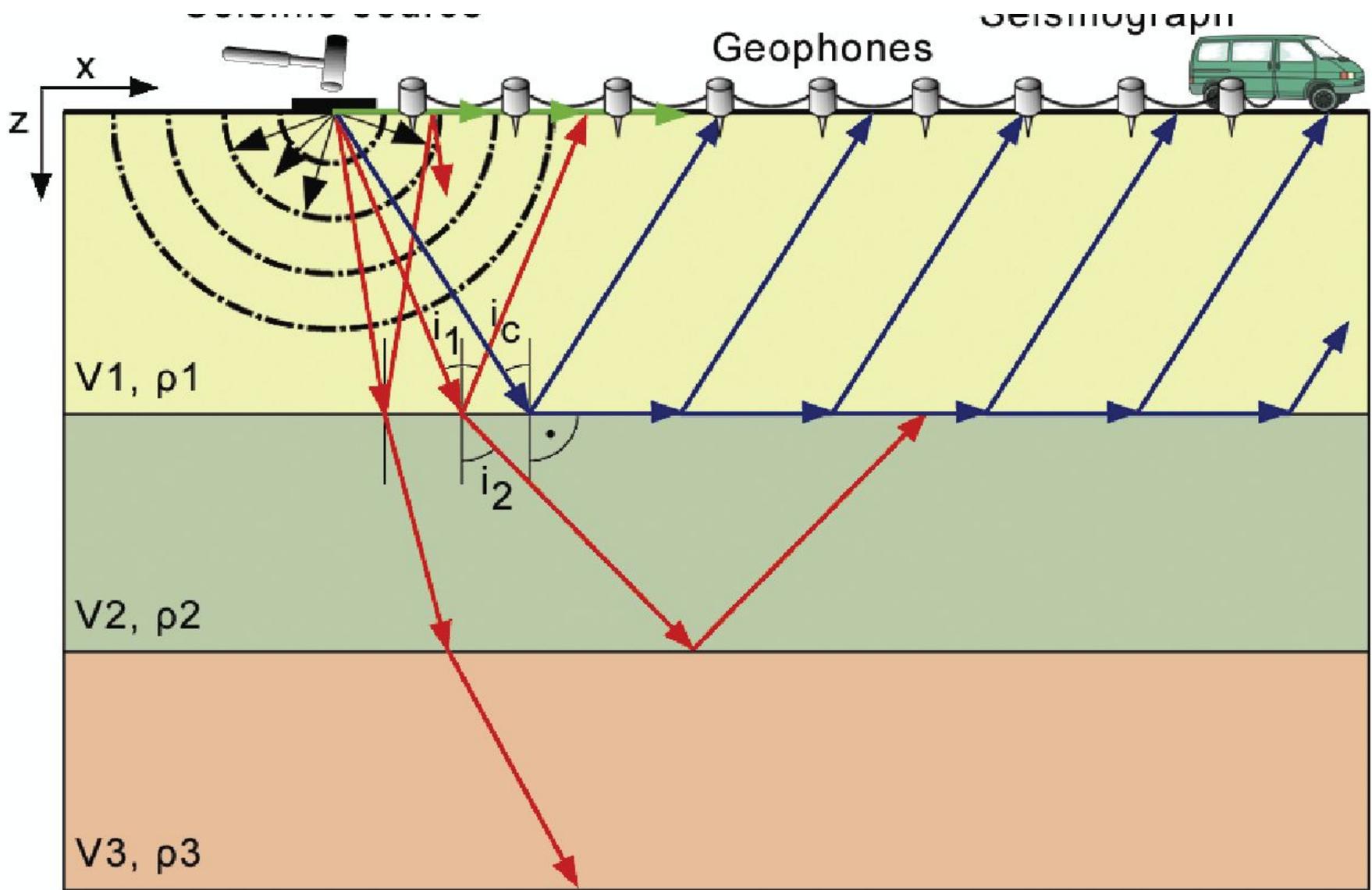
# МПВ

## Метод преломленных волн

Составил

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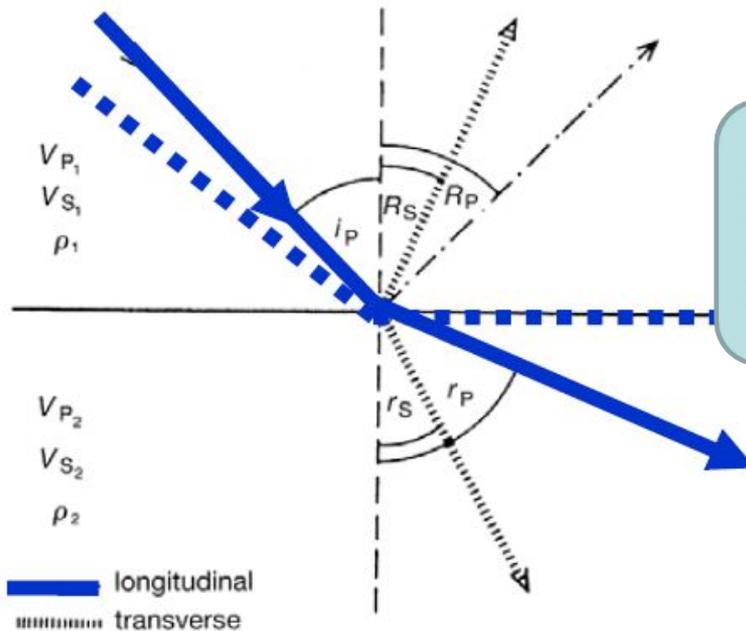


# Критическое падение

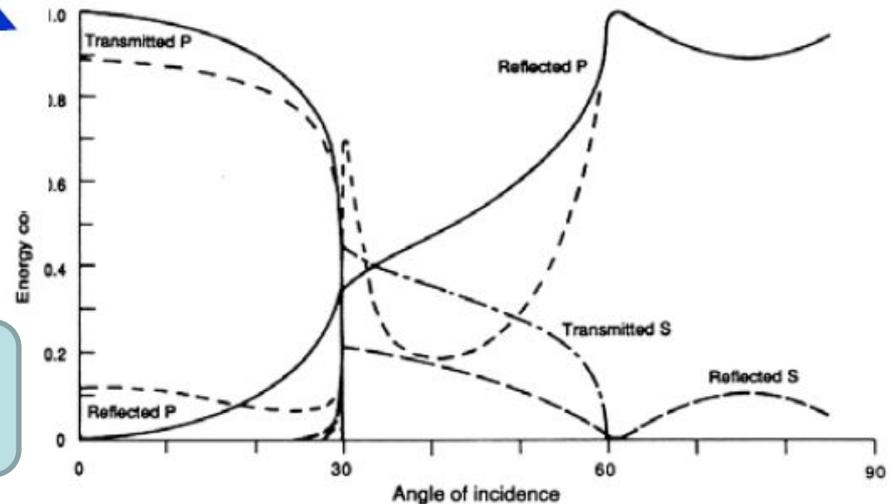
Когда  $r_p = 90^\circ$   $i_p = i_c$  Критический угол равен

$$\sin i_c = \frac{V_{P1}}{V_{P2}}$$

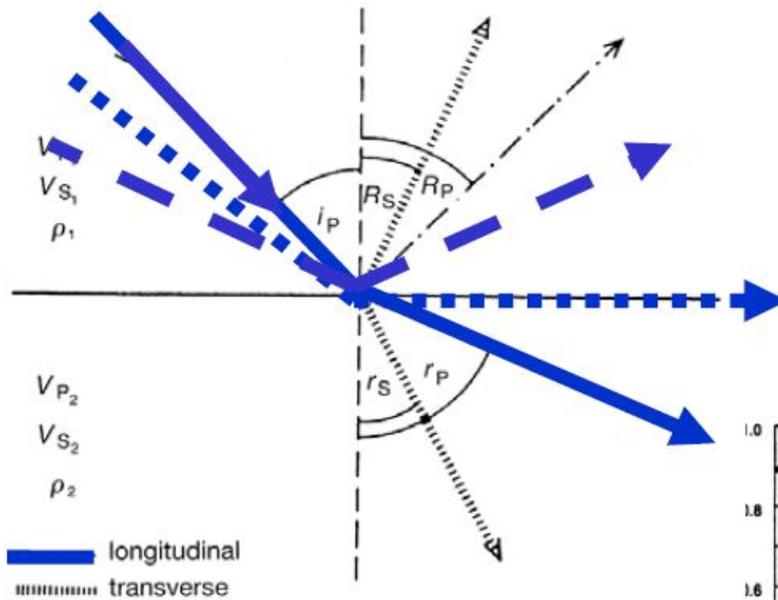
Волна упавшая под критическим углом на границу, распространяется вдоль не со скоростью  $V_2$  в нижнем слое.  $V_2 > V_1$



# Коэффициенты отражения-преломления

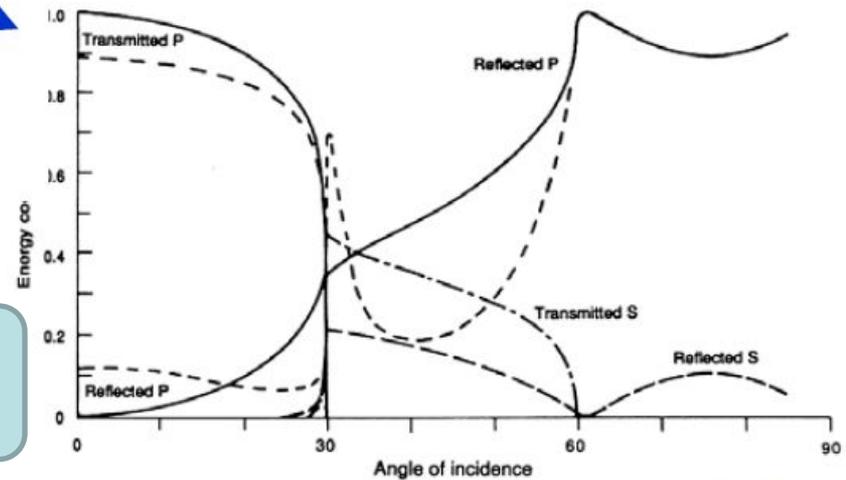


## Закритическое падение



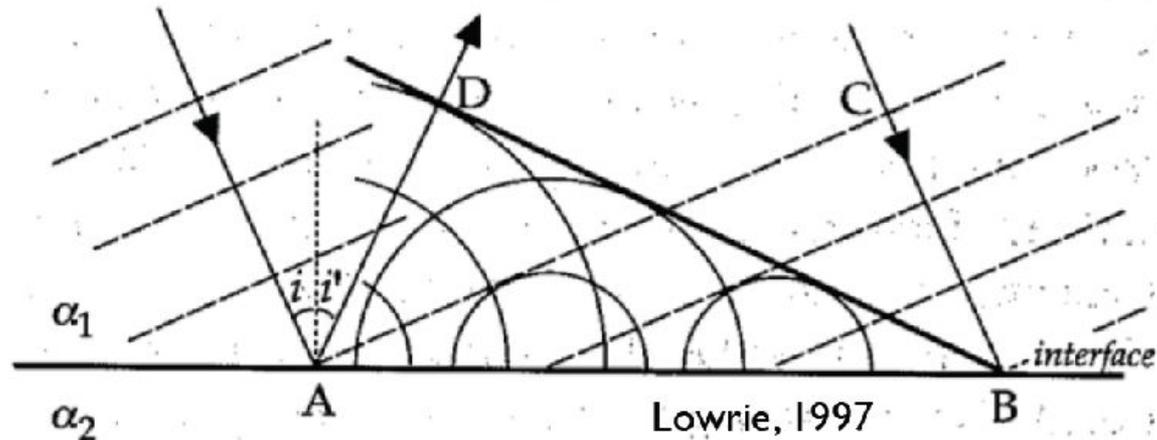
Когда угол падения  $> i_c$  вся энергия волны отражается.

## Коэффициенты отражения-преломления



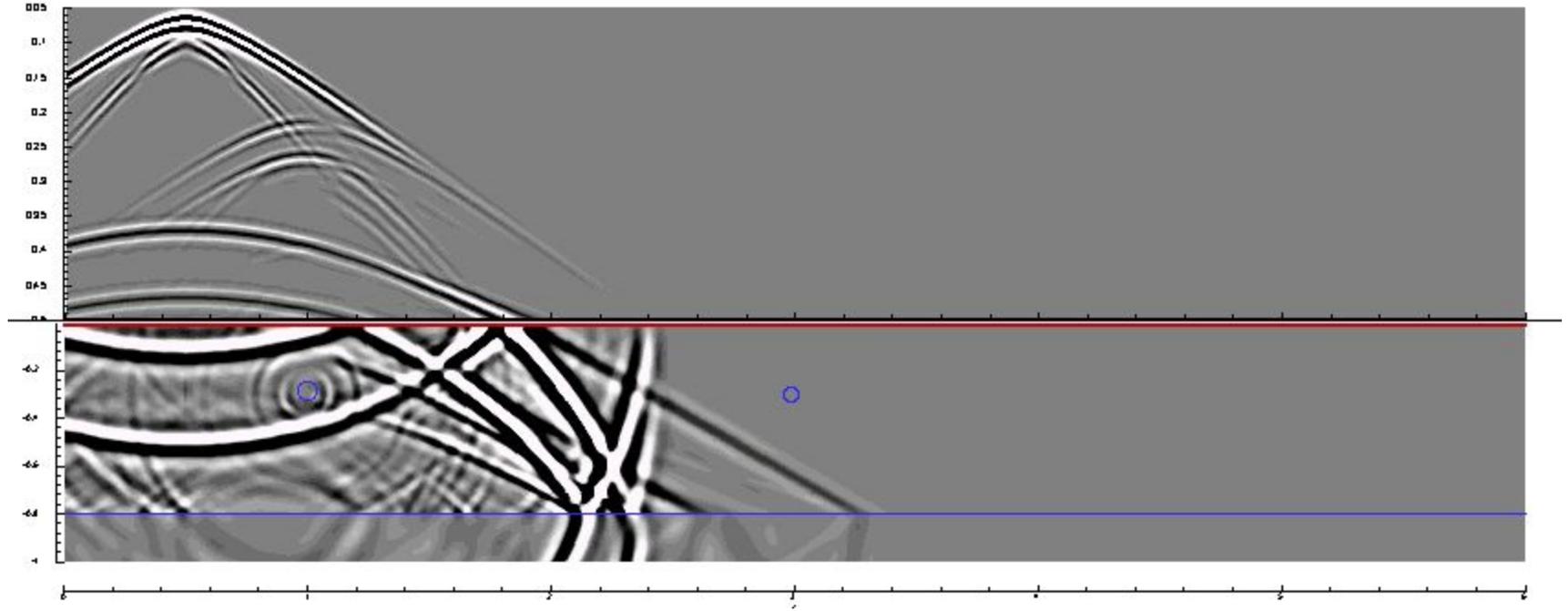
# The law of reflection using Huygens wavelets

The angle of reflection is equal to the angle of incidence.



- During the time that wavefront  $AC$  travels from intersecting at  $A$  to intersecting at  $B$ , secondary waves travel from  $A$  to  $D$ .
- $AD = CB$ , so angles are also equal.

# Волновое поле



# Пример

2

Горизонтальная граница

Уравнение  
годографа прямой :

$$T = \frac{x}{V_1}$$

И головной волны:

$$T = T_{SB} + T_{DD'} + T_{BD}$$

$$T = \frac{2h_1}{V_1 \cos i_c} + \frac{x - 2h_1 \tan i_c}{V_2}$$

$$T = \frac{x}{V_2} + \frac{2h_1 \sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$

$$T = ax + b$$

Наклон определяется  
скоростью, глубина  
точкой пересечения.

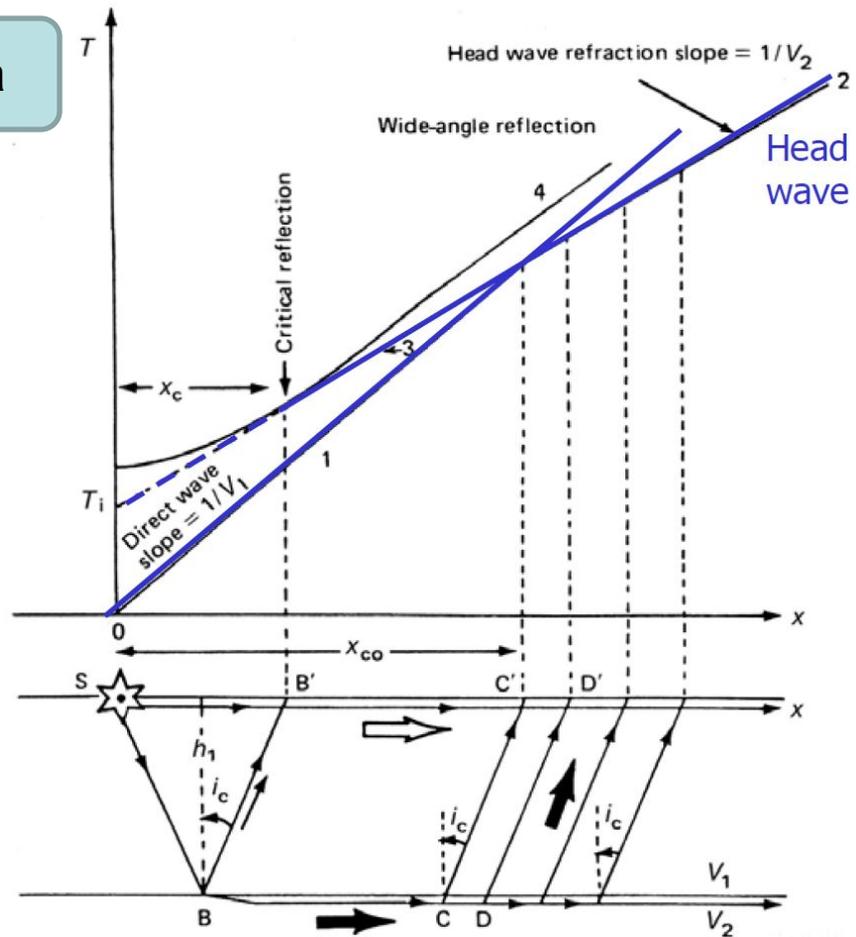


Fig. 4.34 Principle of the seismic refraction method. Travel-time curves for direct waves, critically refracted waves (head waves), late refracted arrivals, and reflected waves are shown by numbers 1-4, respectively. Note that critically refracted waves start arriving after a critical distance  $x_c$ , but they overtake the direct waves at a crossover distance  $x_{co}$ .

Горизонтальная граница

Точка пересечения  
 годографов головной и  
 прямой волны  
 определяется:

$$\frac{x_{co}}{V_1} = \frac{x_c}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2V_1}$$

$$x_{co} = 2h_1 \frac{\sqrt{V_2 + V_1}}{\sqrt{V_2 - V_1}}$$

-альтернативный подход к  
 определению мощности

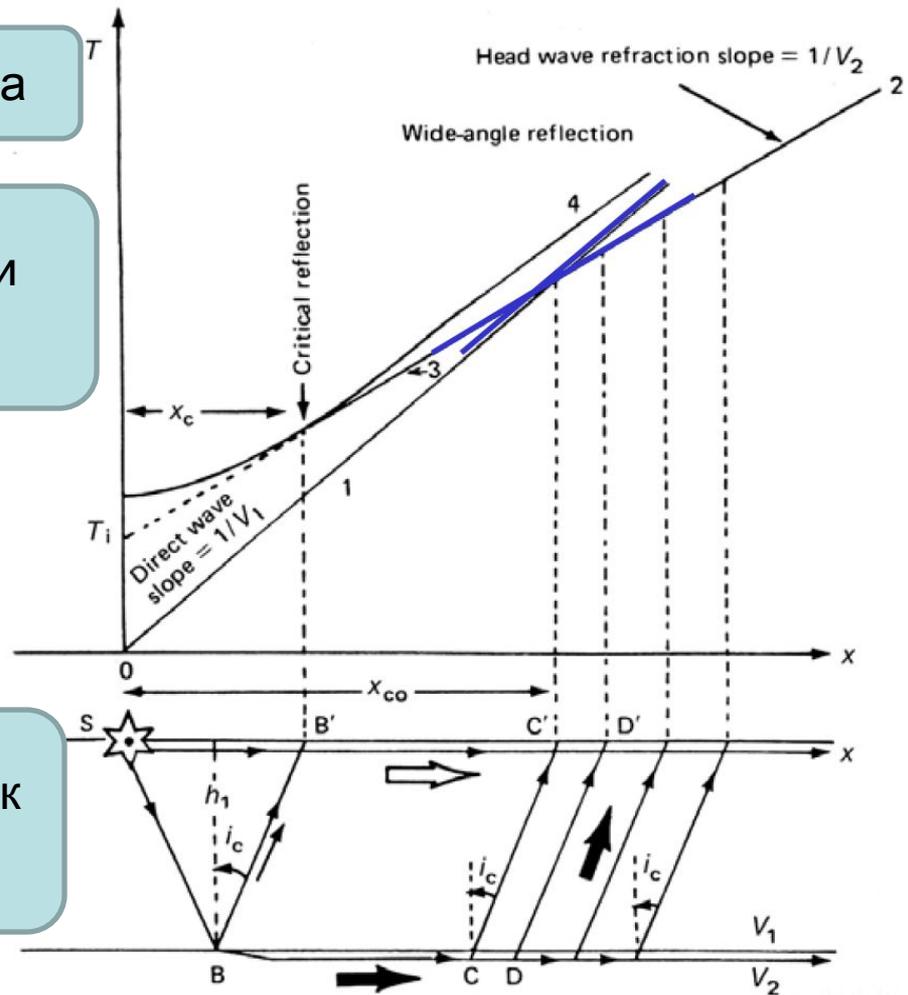


Fig. 4.34 Principle of the seismic refraction method. Travel-time curves for direct waves, critically refracted waves (head waves), late refracted arrivals, and reflected waves are shown by numbers 1-4, respectively. Note that critically refracted waves start arriving after a critical distance  $x_c$ , but they overtake the direct waves at a crossover distance  $x_{co}$ .

# Horizontal interface

## Crossover distance, $x_{co}$

Where the direct and head wave cross. Their travel times are equal:

$$\frac{x_{co}}{V_1} = \frac{x_{co}}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2V_1}$$

$$x_{co} = 2h_1 \frac{\sqrt{V_2 + V_1}}{\sqrt{V_2 - V_1}}$$

Another approach to obtaining layer thickness

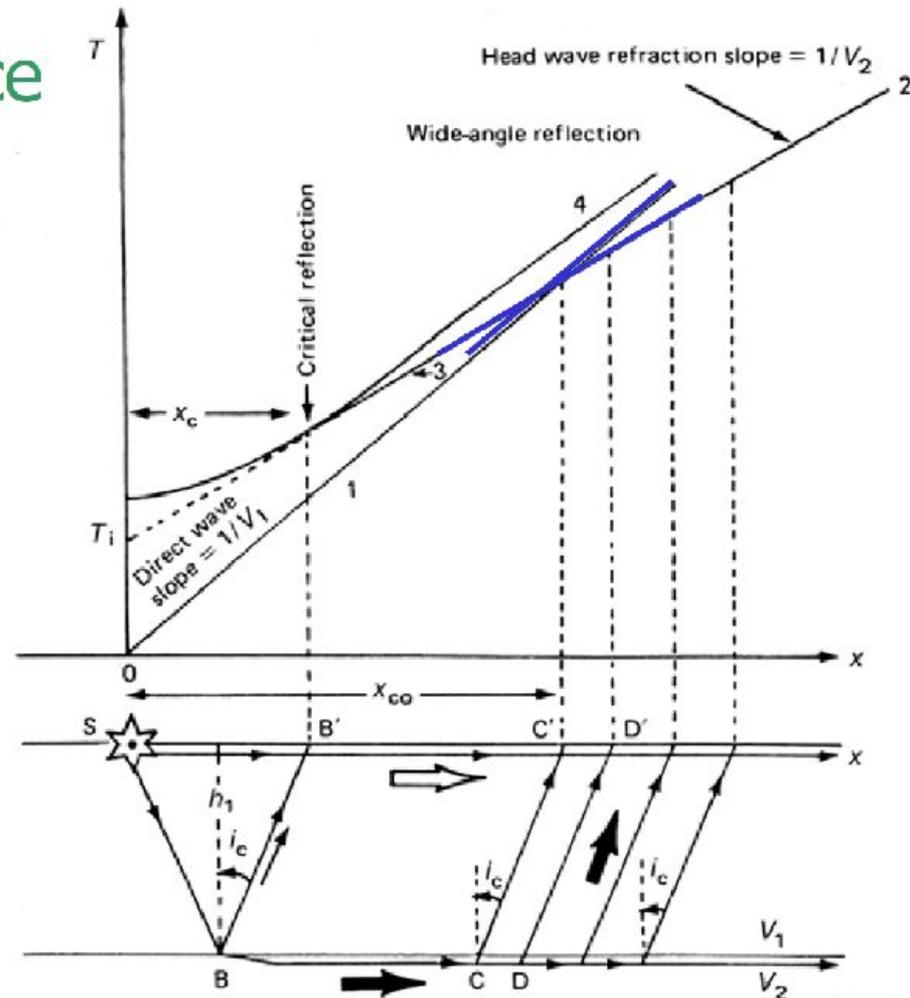


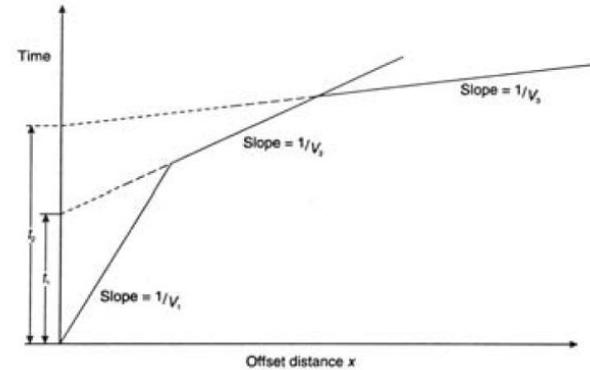
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## Трёхслойная модель

Traveltime

$$T_{SG} = \frac{SA}{V_1} + \frac{AB}{V_2} + \frac{BC}{V_3} + \frac{CD}{V_2} + \frac{DG}{V_1}$$

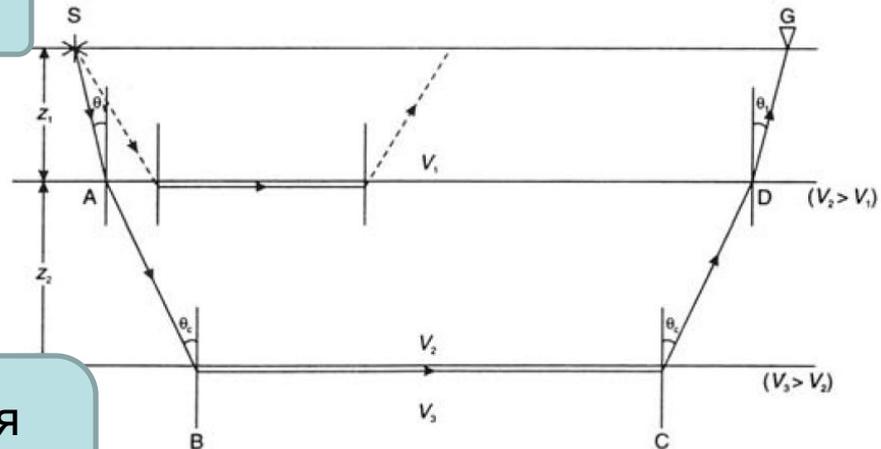
$$T_{SG} = \frac{2z_1}{V_1 \cos \theta_1} + \frac{2z_2}{V_2 \cos \theta_c} + \frac{x - 2z_1 \tan \theta_1 - 2z_2 \tan \theta_c}{V_3}$$



После преобразования

$$T_{SG} = \frac{x}{V_3} + \frac{2z_1 \sqrt{V_3^2 - V_1^2}}{V_3 V_1} + \frac{2z_2 \sqrt{V_3^2 - V_2^2}}{V_3 V_2}$$

$V_1$   $V_2$   $V_3$  из кажущихся скоростей, глубины по точкам пересечения.



# Multiple-layered models

For multiple layered models we can apply the same process to determine layer thickness and velocity sequentially from the top layer to the bottom

Head wave from top of layer 2:

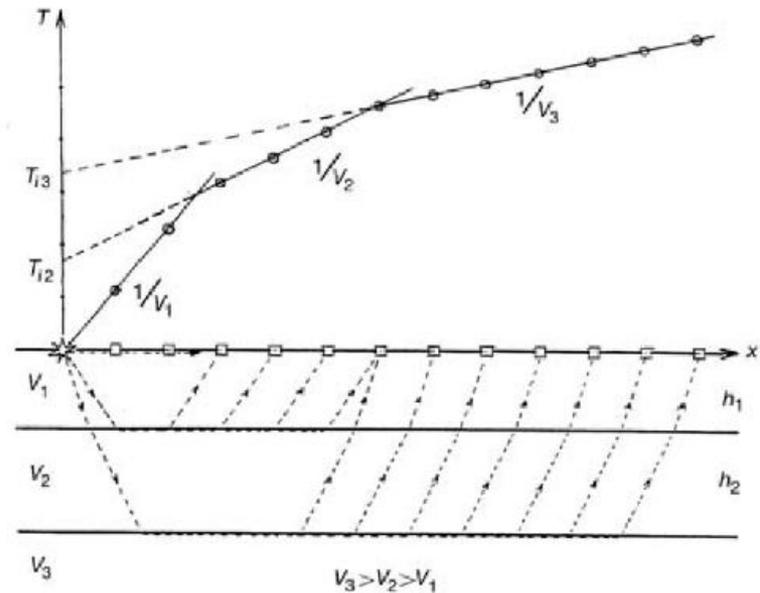
$$T = \frac{x}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2V_1}$$

Head wave from top of layer 3:

$$T = \frac{x}{V_3} + \frac{2h_1\sqrt{V_3^2 - V_1^2}}{V_3V_1} + \frac{2h_2\sqrt{V_3^2 - V_2^2}}{V_3V_2}$$

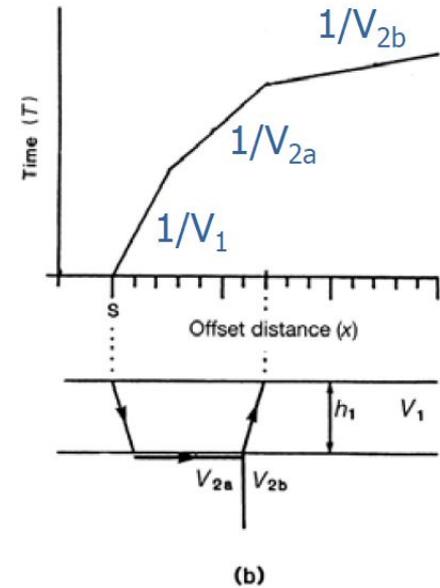
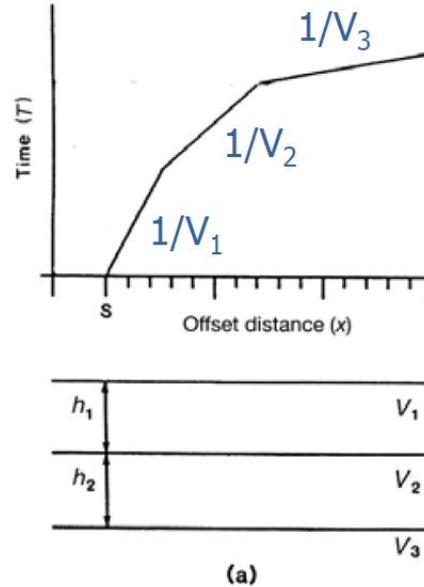
Head wave from top of layer n:

$$T = \sum_{j=1}^{n-1} \left( \frac{2h_j\sqrt{V_n^2 - V_j^2}}{V_nV_j} \right) + \frac{x}{V_n}$$



## Горизонтальный и вертикальный контраст скоростей

Кривизна единичного годографа головных волн может быть описана как трехслойной средой так и двухслойной, с вариацией скорости во втором слое.



Head wave  
continues into 2b

## Горизонтальный и вертикальный контраст скоростей

Сравнение и различие влияния на годограф преломленной волны вертикальной и горизонтальной неоднородности разреза.

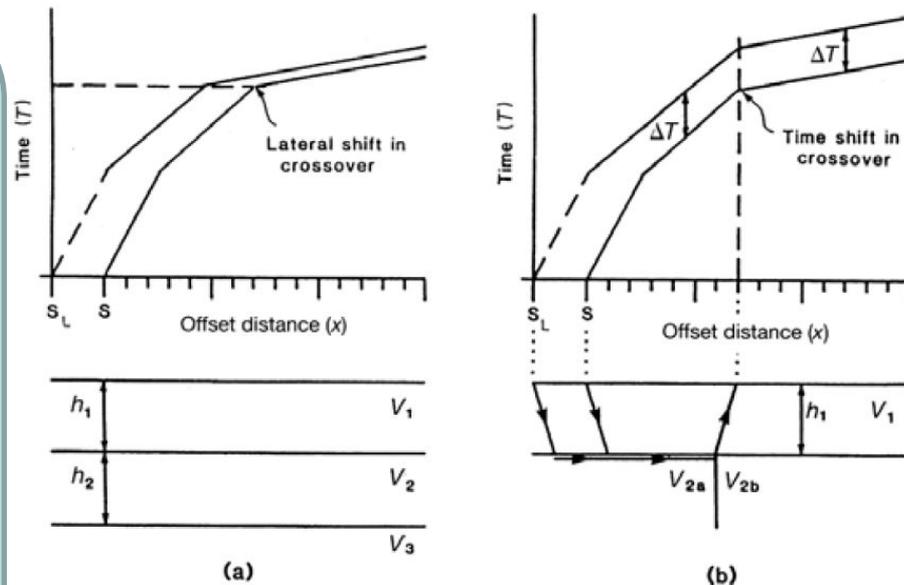


Fig. 4.36 Identical travel-time curves arising from a single shot  $S$  over (a) a horizontal three-layer earth, and (b) a two-layer earth with lateral velocity change in the second layer. The ambiguity can be resolved by an additional long offset shot,  $S_L$ . Dashed lines on long offset travel-time curves are segments of the travel-time curves that would be observed if geophones were placed between  $S$  and  $S_L$ . The three-layer case (a) can be distinguished from the two-layer case (b) by noting a lateral shift in the crossover point in the first case and a vertical shift in the crossover point in the second case. (Modified from Lankston, 1990.)

# Mapping vertical contacts

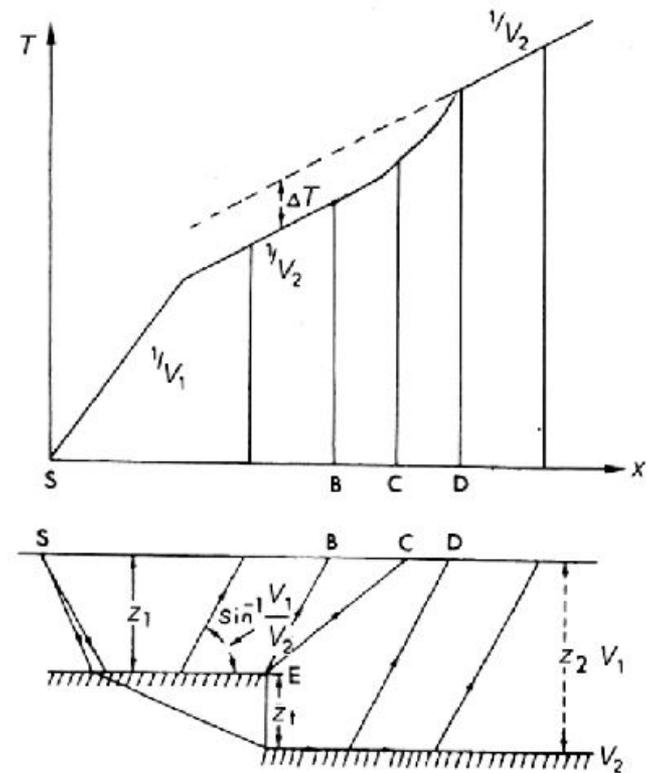
## Small offsets

### A vertical step causes an offset on the travelttime curve

- The relation of velocity to the slope remains unchanged
- The offset can be calculated from the time offset,  $\Delta T$

$$z_t = \frac{\Delta T V_2 V_1}{\sqrt{V_2^2 - V_1^2}}$$

- Diffractions link the two head wave curves
- Depth,  $z_1$ , is calculated from the intercept in the usual way



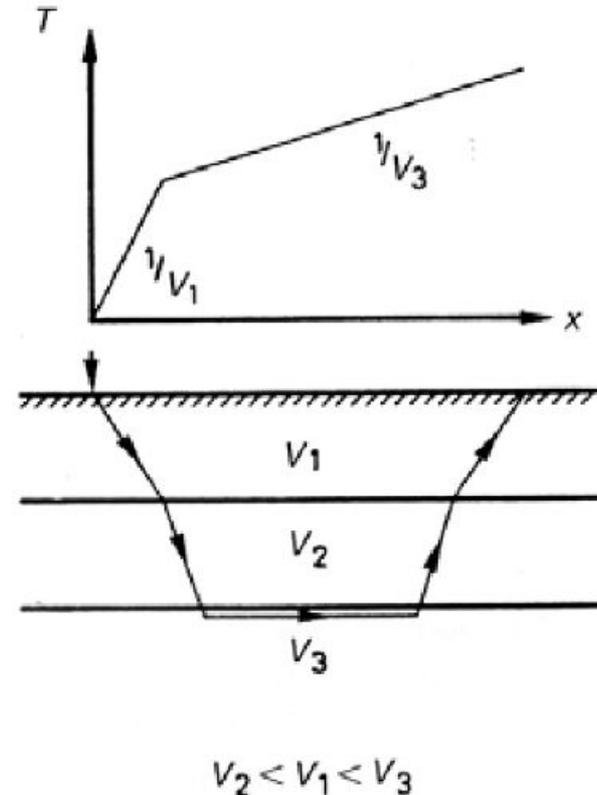
## Limitations

# Low velocity layers

- They are completely invisible to the refraction method
- They will cause miss-interpretation of the depth of lower lying layers

The intercept of the refraction from layer 3 will be dependent on the thickness and velocity in layer 2

- Lower layers appear deeper than they are



Годограф Преломленной волны для случая наклонной границы.

$$T_d = \frac{SC}{V_1} + \frac{CD}{V_2} + \frac{DS'}{V_1}$$

$$T_d = \frac{h_u + h_d}{V_1 \cos i_c} + \frac{x - [(h_u + h_d) \tan i_c]}{V_2}$$

После тригонометрических преобразований получим

Ф-ла Годографа по падению и кажущаяся скорость

$$T_d = \frac{x \sin(i_c + \phi)}{V_1} + \frac{2h_d \cos i_c}{V_1}$$

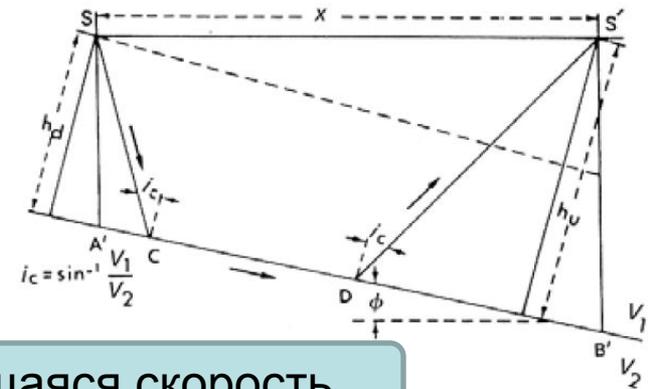
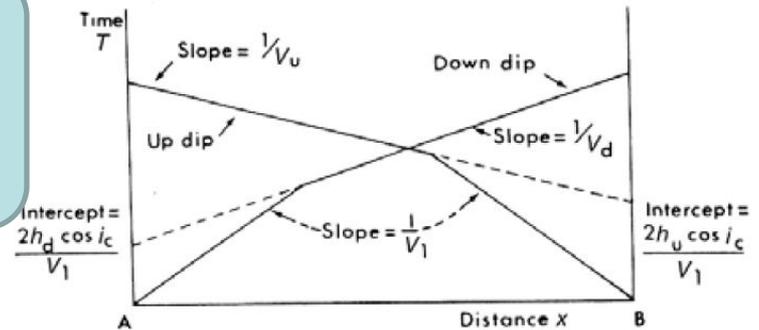
$$V_d = \frac{V_1}{\sin(i_c + \phi)}$$

Ф-ла Годографа по восстанию и кажущаяся скорость

$$T_u = \frac{x \sin(i_c - \phi)}{V_1} + \frac{2h_u \cos i_c}{V_1}$$

$$V_u = \frac{V_1}{\sin(i_c - \phi)}$$

Кстати...где V2?



# Dipping layer traveltimes

Given

$$V_d = \frac{V_1}{\sin(i_c + \phi)} \quad V_u = \frac{V_1}{\sin(i_c - \phi)}$$

We can solve for:

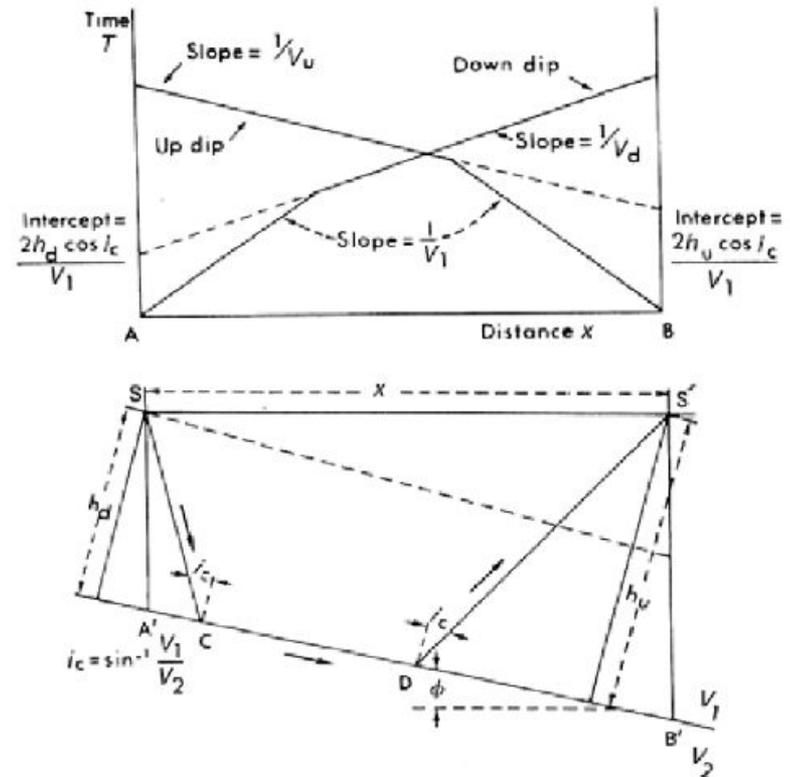
$$\phi = \frac{1}{2} \left[ \sin^{-1} \frac{V_1}{V_d} - \sin^{-1} \frac{V_1}{V_u} \right]$$

$$i_c = \frac{1}{2} \left[ \sin^{-1} \frac{V_1}{V_d} + \sin^{-1} \frac{V_1}{V_u} \right]$$

$V_2$  then obtained from:  $\sin i_c = \frac{V_1}{V_2}$

Finally, the intercept times can be used to determine the perpendicular distance to the reflector:

$$T_{id} = \frac{2h_d \cos i_c}{V_1} \quad T_{iu} = \frac{2h_u \cos i_c}{V_1}$$



# Real Earth "flat" layers

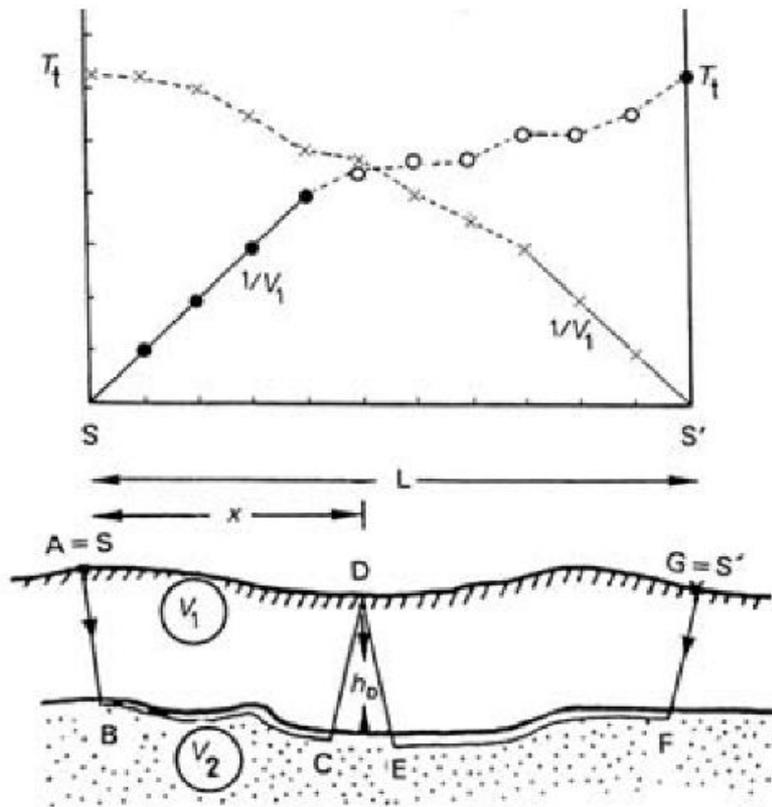
Although the interfaces between real Earth layers are not perfectly flat, head waves still travel along them

Analysis methods:

**Best-fit straight line** through the points provides some kind of average layer thickness and velocity though the error may be unacceptable

Special analysis techniques:

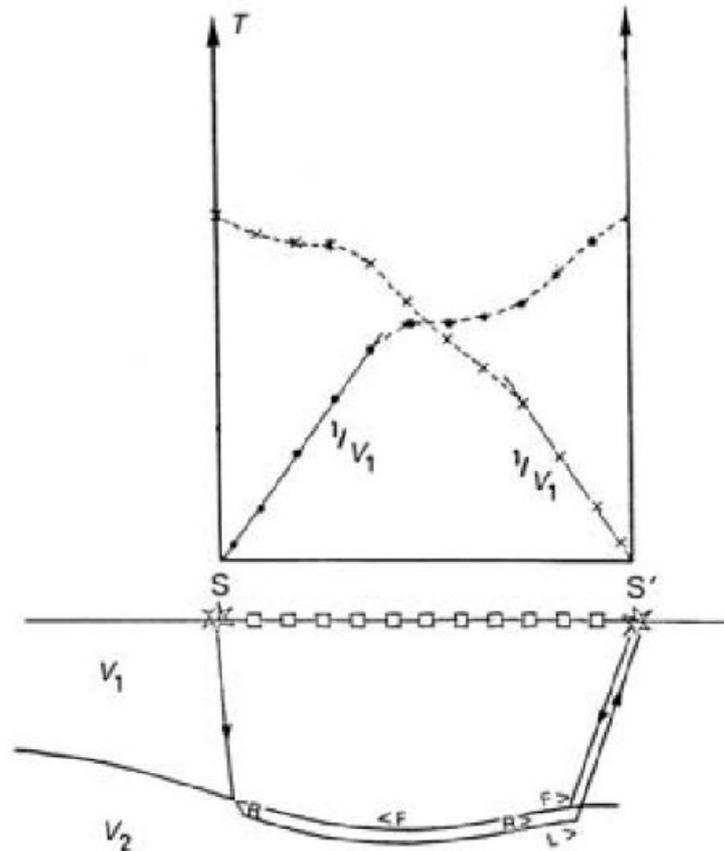
- Phantom arrivals**
- Delay time**
- Plus-minus**



# Phantom arrivals

## Undulating surface

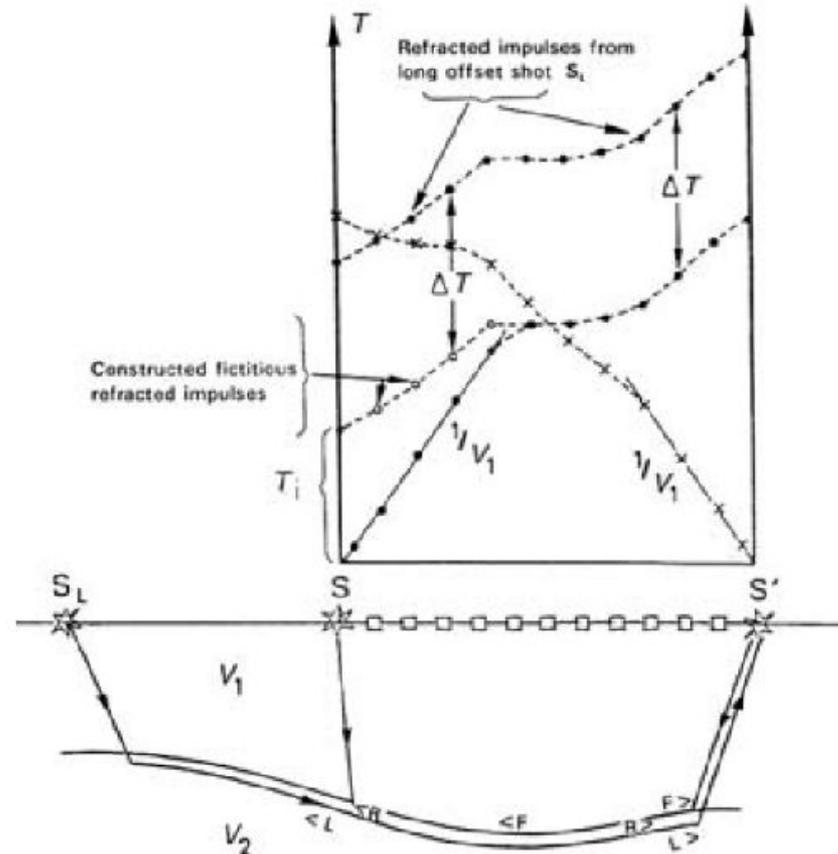
- Cannot extrapolate the head wave arrival time curve back to the intercept
- How do we determine layer thickness beneath the shot,  $S$ ?



# Phantom arrivals

1. Shoot a long-offset shot,  $S_L$
2. The head wave traveltime curves for both shots will be parallel, offset by time  $\Delta T$
3. Subtract  $\Delta T$  from the  $S_L$  arrivals to generate fictitious 2<sup>nd</sup> layer arrivals close to  $S$  – the phantom arrivals
4. The intercept point at  $S$  can then be determined:  $T_i$
5. Use the usual formula to determine perpendicular layer thickness beneath  $S$

$$T_i = \frac{2h_s \sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$



# Separation of delay times

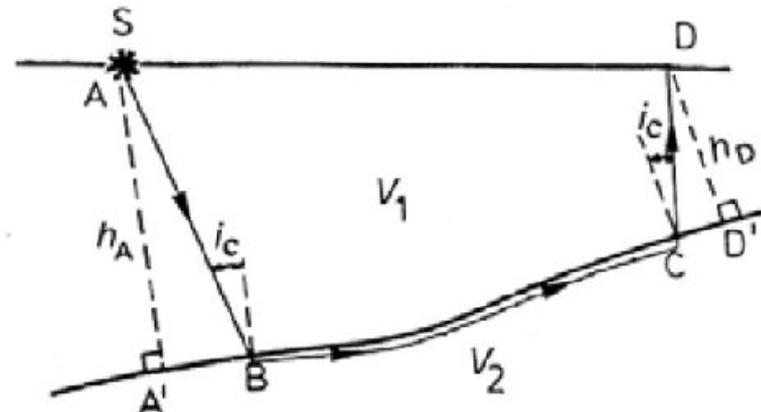
**Delay time** is the difference between the slant path traveltime (AB) and the traveltime along the refractor beneath (A'B)

Delay time below shotpoint S:

$$\Delta T_S = \frac{AB}{V_1} - \frac{A'B}{V_2}$$

$$\Delta T_S = \frac{h_A}{V_1 \cos i_c} - \frac{h_A \tan i_c}{V_2}$$

$$\Delta T_S = \frac{h_A \sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$



Likewise, beneath D:

$$\Delta T_D = \frac{h_D \sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$

The delay time is related to the perpendicular depth beneath the geophone. If we know the delay time,  $V_1$  and  $V_2$  then we can calculate the depth.

# Separation of delay times

**Determining the delay time beneath a geophone,  $\Delta T_D$ :**

Total travelttime ABFG:

$$T_t = \frac{L}{V_2} + \Delta T'_S + \Delta T'_{S'}$$

Total travelttime ABCD:

$$T_{SD} = \frac{x}{V_2} + \Delta T'_S + \Delta T'_D$$

Total travelttime GFED:

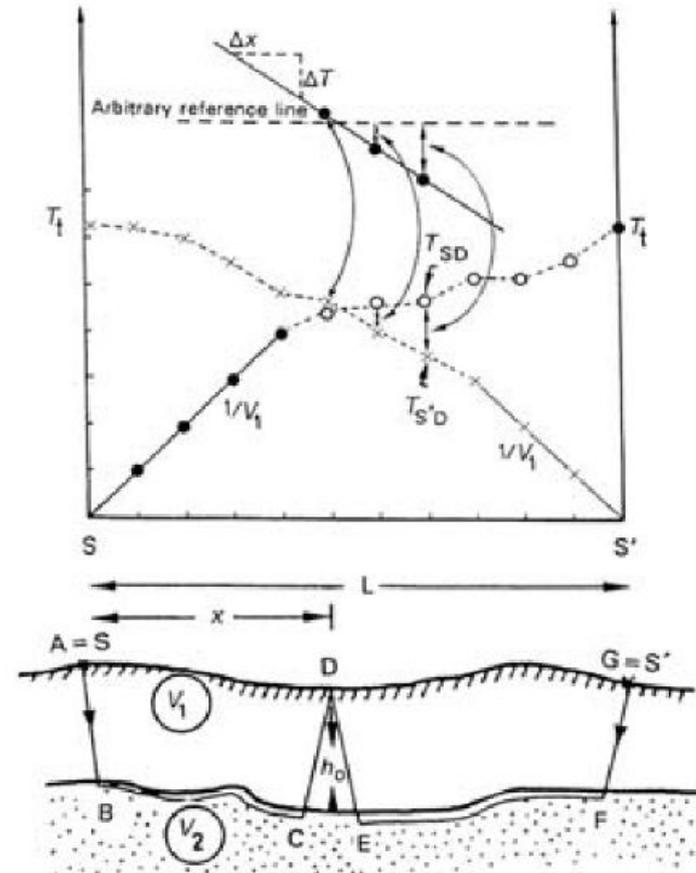
$$T_{S'D} = \frac{L-x}{V_2} + \Delta T'_{S'} + \Delta T'_D$$

**“plus”**

Adding the equations for  $T_{SD}$  and  $T_{S'D}$  and using the equation for  $T_t$  we obtain an equation for  $\Delta T_D$ :

$$\Delta T'_D = \frac{T_{SD} + T_{S'D} - T_t}{2}$$

**→ determine  $\Delta T_D$  from a reversed refraction line**



# Separation of delay times

## Determining velocity, $V_1$ $V_2$ :

Direct arrival slope =  $1/V_1$

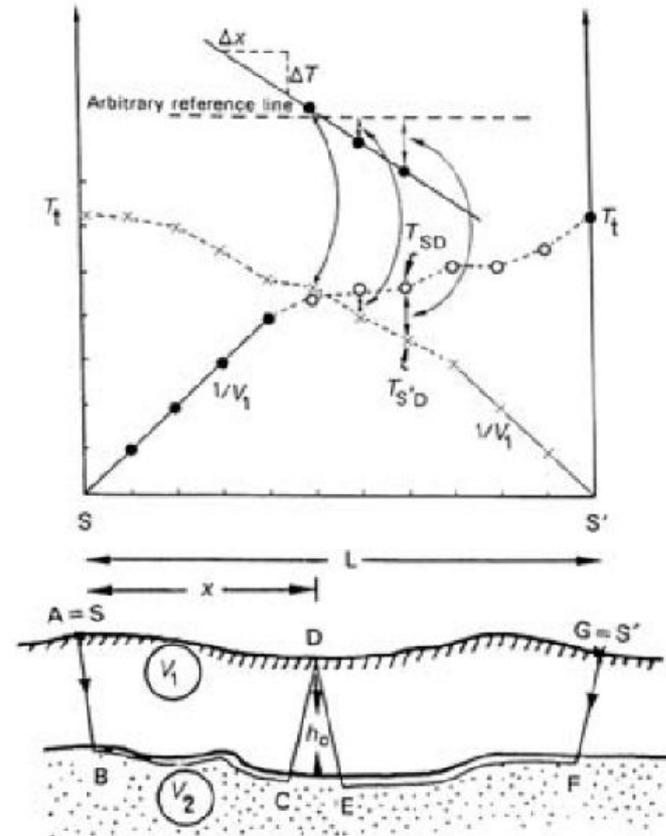
**"minus"**

Subtracting the equations for  $T_{SD}$  and  $T_{S'D}$  we obtain the equation:

$$T_{SD} - T_{S'D} = \frac{2x}{V_2} - \frac{L}{V_2} + \Delta T_S - \Delta T_{S'}$$

$$y = ax + b$$

- Plotting  $(T_{SD} - T_{S'D})$  against  $x$  the slope is  $2/V_2$
- Variations in the slope reflect lateral variations in velocity



# The plus-minus method

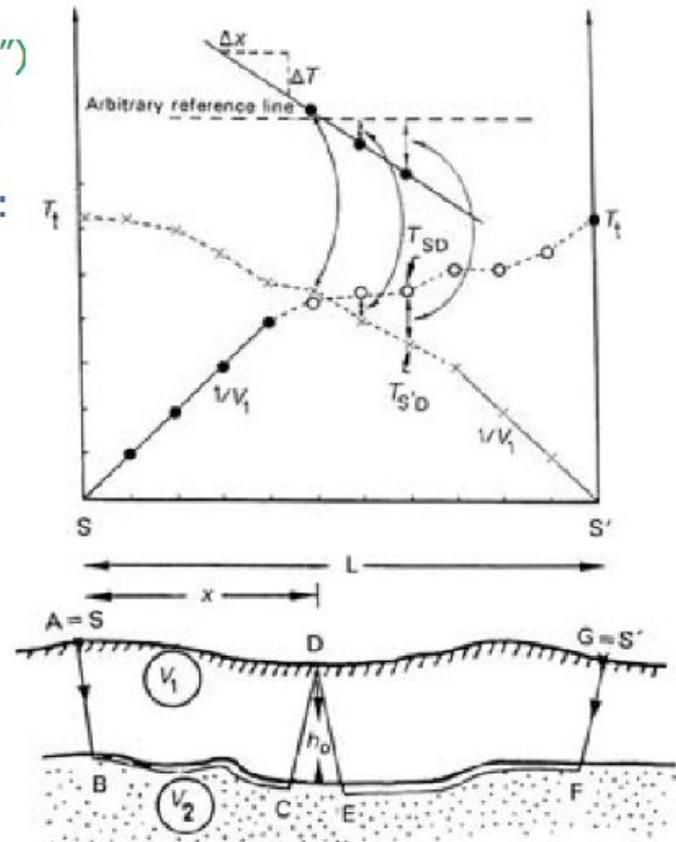
Given the delay time  $\Delta T_D$ , (from the "plus")  
and the velocities  $V_1$  (from direct arrival)  
and  $V_2$  (from the "minus")

We can calculate the perpendicular depth:

$$\Delta T_D = \frac{h_D \sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$

Note: we need to see refracted arrivals  
from both forward and reverse shots  
(three geophones only in figure)

Use long offset shots to collect  
necessary data for all geophones

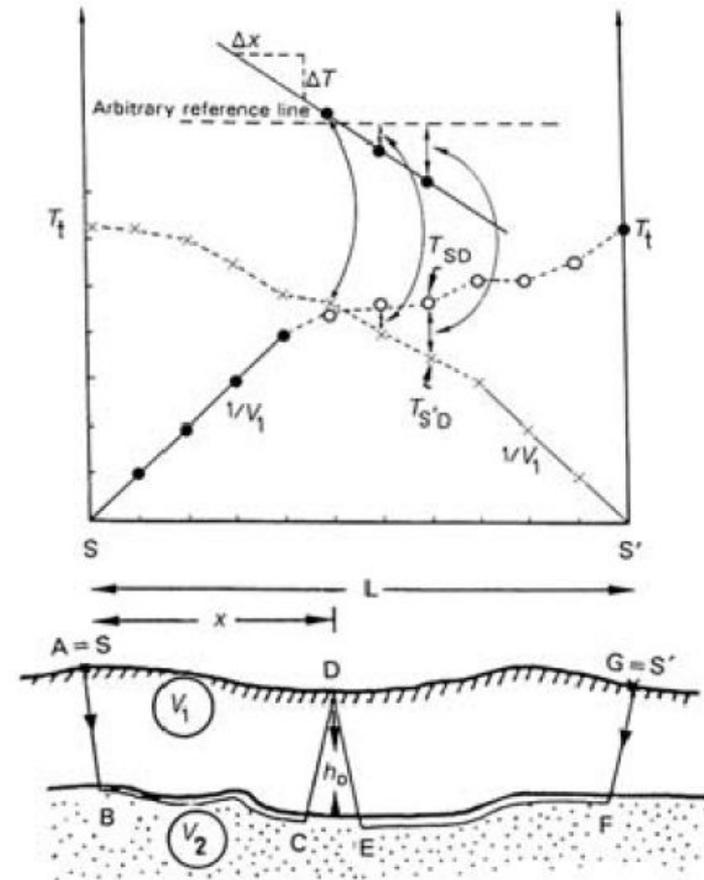


# The plus-minus method

## Assumptions and approximations

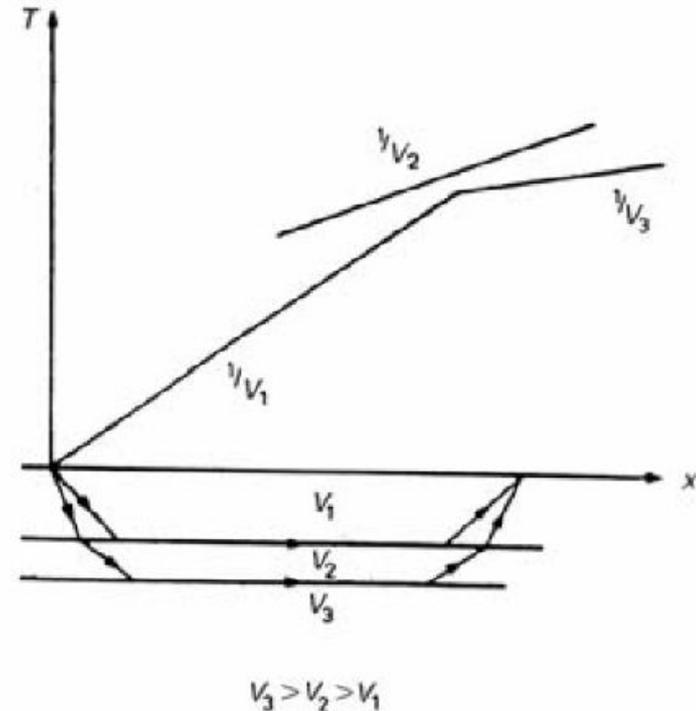
1. The relief on the refractor must be small compared to the depth
2. Geometric relations assume a small refractor dip (dip < 10 deg)
3. Refractor assumed to be planar between the two points of emergence to a given geophone (ie between C and E)

So that  $\Delta T_D$  is equal when shooting from both sides

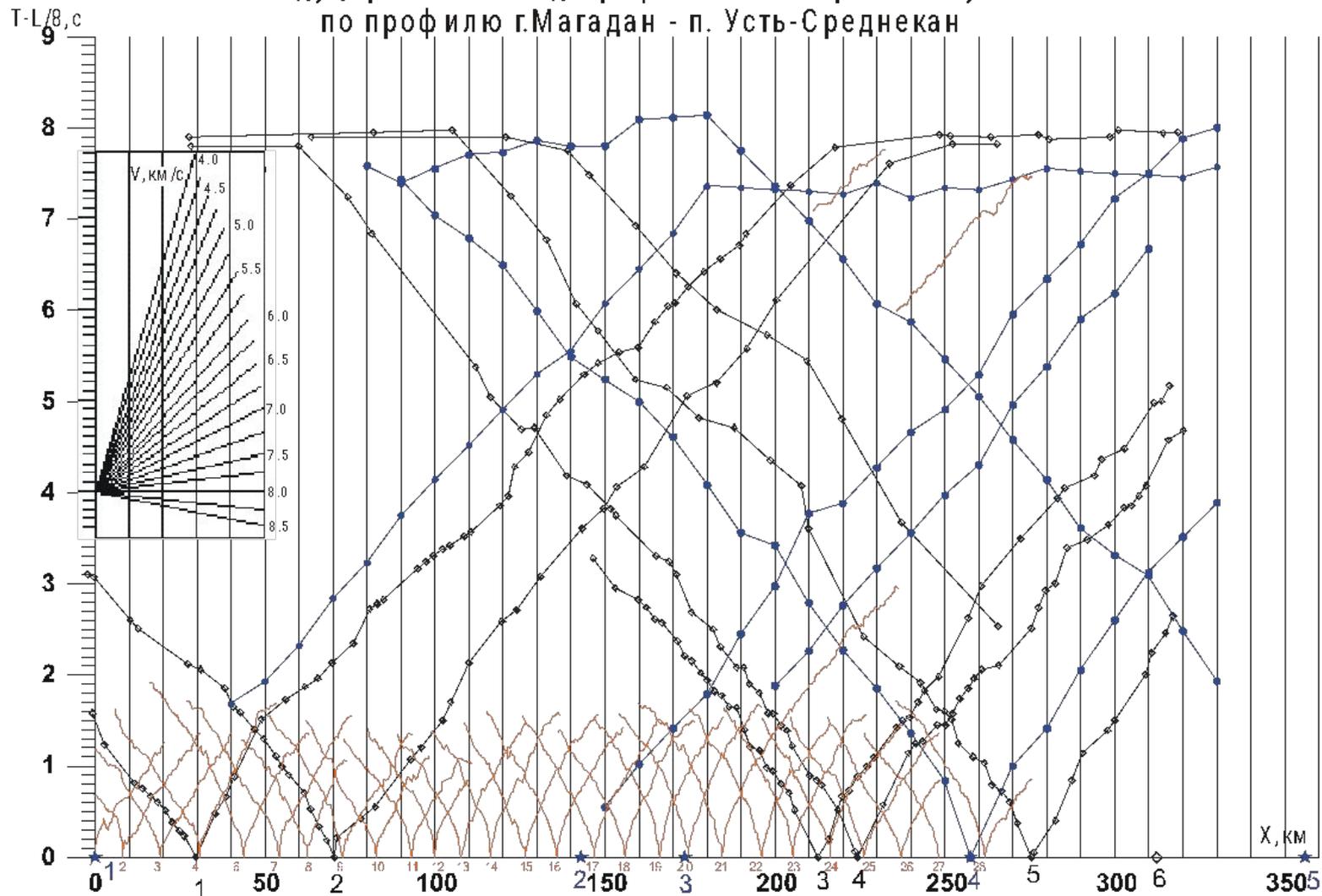


## Limitations Hidden layers

- **If a layer is thin it may never produce a first arrival**  
Either the direct or refraction from a lower (much higher velocity layer) is always first
- **Lower layers always appear too shallow as a layer has been missed**

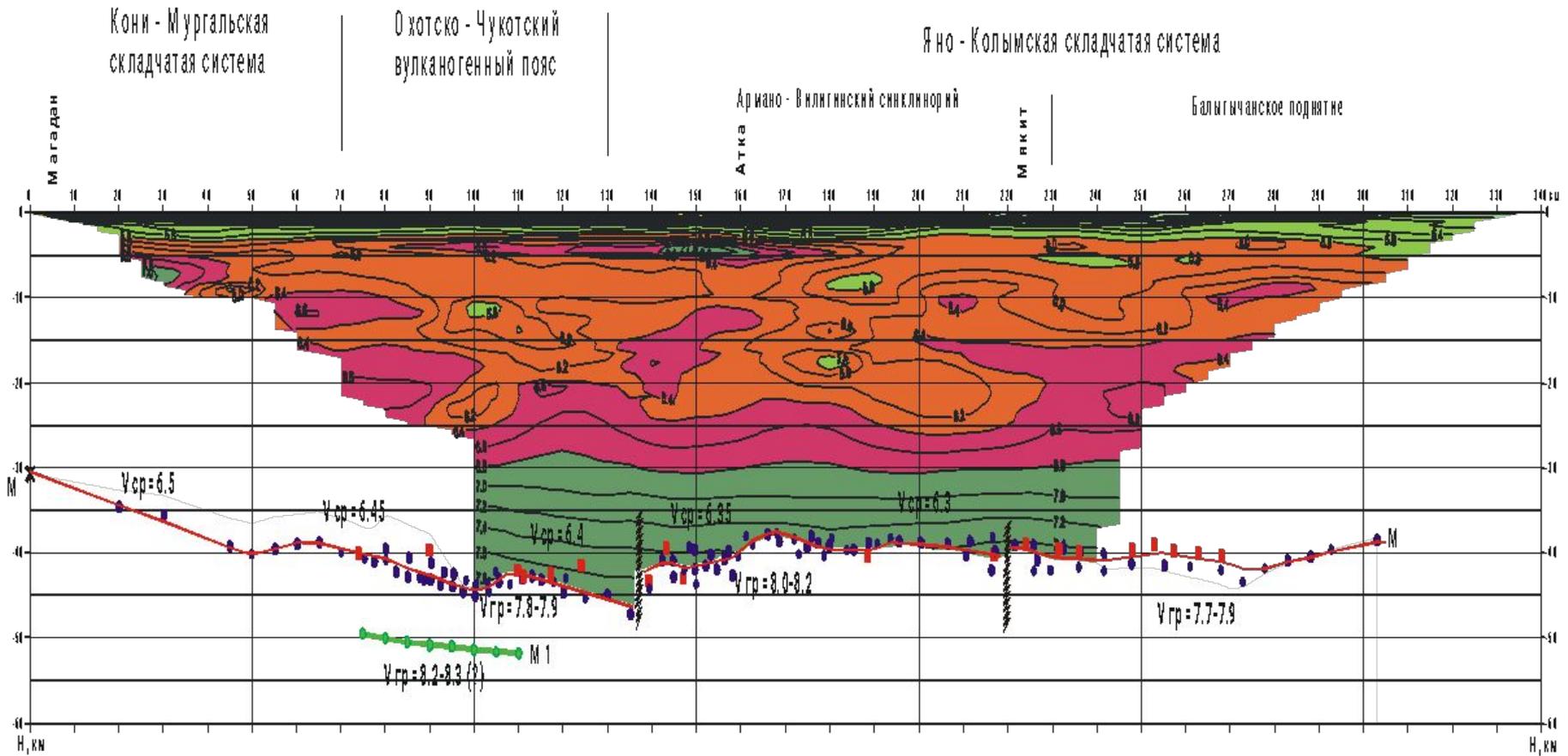


## Редуцированные годографы волн в первых вступлениях по профилю г.Магадан - п. Усть-Среднекан



- Данные ГСЗ с вибратором 2002г
- Данные ГСЗ 1959г
- Данные КМПВ и ГСЗ 2001г.

# Сейсмотографический разрез по Южно-Магаданскому участку профиля 2-ДВ (по материалам ГСЗ и КМПВ)



## Условные обозначения

- Глубины до поверхности Мохоровичича по данным отраженных волн (преломленных волн, с карты рельефа М (морские работы 50-х годов))
- Глубины до поверхности M1(?) по данным широкоугольных отраженных волн
- Граница М
- Граница M1
- $V_{ср}, V_{гр}$  - значения средних и граничных скоростей в км/с
- Слои консолидированной коры:
- Гранитоидный
  - Гранулитовый
  - Базальтовый
- Зоны изменения граничных скоростей по поверхности Мохоровичича (возможные зоны глубинных разломов)