Решение пределов

1) $$\lim_{x\to -2}\frac{x^4+2}{3x^2-1}=\frac{16+2}{3\cdot 4-1}=\frac{18}{11}=1\frac{7}{11}$$

2) $$\lim_{x\to -1}\frac{x^3+1}{x+1}=\left[\frac{0}{0}\right]=\lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{x+1}=\lim_{x\to -1}(x^2-x+1)=3$$

3) $$\lim_{x\to -3}\frac{x+3}{\sqrt{x+4}-1}=\left[\frac{0}{0}\right]=\lim_{x\to -3}\frac{(x+3)(\sqrt{x+4}+1)}{(\sqrt{x+4}-1)(\sqrt{x+4}+1)}=\lim_{x\to -3}\frac{(x+3)(\sqrt{x+4}+1)}{\left(\sqrt{x+4}\right)^2-1^2}=$$

a - b a + b

$$=\lim_{x\to -3}\frac{(x+3)(\sqrt{x+4}+1)}{x+3}=\lim_{x\to -3}(\sqrt{x+4}+1)=\sqrt{-3+4}+1=2$$

$$4)\ \lim_{x\to -1}\frac{3x^2+2x-1}{2x^2+x-1}=[\frac{0}{0}]=\lim_{x\to -1}\frac{3(x+1)(x-\frac{1}{3})}{2(x+1)(x-\frac{1}{2})}=\lim_{x\to -1}\frac{3x-1}{2x-1}=\frac{-3-1}{-2-1}=\frac{-4}{-3}=1\frac{1}{3}$$

$$3x^2+2x-1=3(x-\frac{1}{3})(x+1)$$

$$D=4-4\cdot 3\cdot(-1)=16\qquad x_1=\frac{1}{3}\quad x_2=-1$$

2 способ: По правилу Лопиталя

$$5)\ \lim_{x\to -1}\frac{3x^2+2x-1}{2x^2+x-1}=\lim_{x\to -1}\frac{(3x^2+2x-1)'}{(2x^2+x-1)'}=\lim_{x\to -1}\frac{6x+2}{4x+1}=\frac{-6+2}{-4+1}=\frac{-4}{-3}=1\frac{1}{3}$$

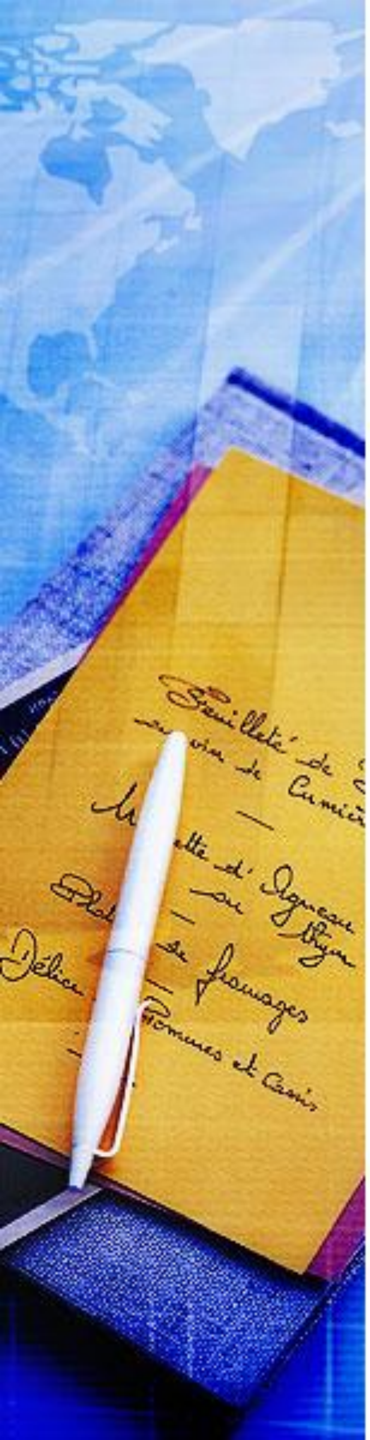
$$6)\ \lim_{x\to\infty}\frac{3x^4-9x^2+8}{2x^4-3x^2-x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{\cancel{x^4}\left(3-\frac{9x^2}{x^4}+\frac{8}{x^4}\right)}{\cancel{x^4}\left(2-\frac{3x^2}{x^4}-\frac{x}{x^4}\right)}=$$

$$=\lim_{x\to\infty}\frac{3-\frac{9}{x^2}+\frac{8}{x^4}}{2-\frac{3}{x^2}-\frac{1}{x^3}}=\lim_{x\to\infty}\frac{3}{2}=\frac{3}{2}=1{,}5$$

(each of $\frac{9}{x^2}$, $\frac{8}{x^4}$, $\frac{3}{x^2}$, $\frac{1}{x^3}$ → 0)

$$6)\ \lim_{x\to\infty}\frac{3x^4-9x^2+8}{2x^4-3x^2-x}\ \square\ \lim_{x\to\infty}\frac{3\cancel{x^4}}{2\cancel{x^4}}=\frac{3}{2}=1{,}5$$

$$\lim_{x\to\infty}\frac{ax^n+.......}{bx^m+.......}=\begin{cases}\dfrac{a}{b}, \text{ если n=m}\\ \infty, \text{ если n>m}\\ 0, \text{ если n<m}\end{cases}$$



$$7)\ \lim_{x\to 2}\frac{5}{4x-8}=\left[\frac{5}{0}\right]=\infty$$

$$\lim_{x\to 2}(4x-8)=0 \Rightarrow$$ **4x – 8 – бм** $\Rightarrow$

$\dfrac{1}{4x-8}$ - бб

$\dfrac{5}{4x-8}$ - бб

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$

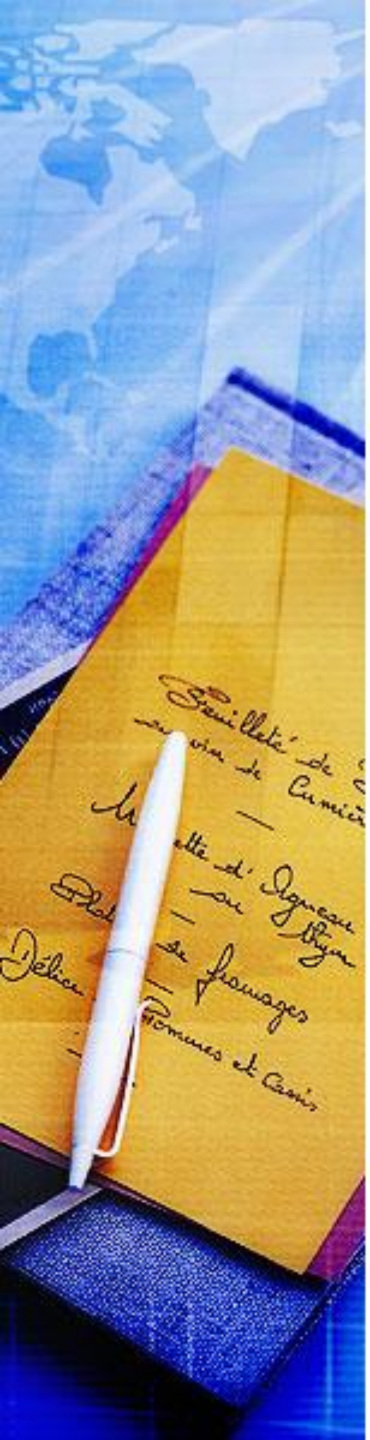
$$1-\cos x=2\sin^2\frac{x}{2}$$

$$8)\ \lim_{x\to 0}\frac{1-\cos x}{x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}=2\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{x}\right)^2=$$

$$=2\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{x}\right)^2=2\left(\lim_{x\to 0}\frac{x/2}{x}\right)^2=2\left(\lim_{x\to 0}\left(\frac{x}{2x}\right)\right)^2=2\cdot\left(\frac{1}{2}\right)^2=\frac{1}{2}$$

$$9)\ \lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{3x}=\left[\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{x}{1}}\right]^3=(e^{\frac{5}{1}})^3=e^{15}$$

$$\lim_{y\to\infty}\left(1+\frac{\alpha}{y}\right)^{\frac{y}{\beta}}=e^{\frac{\alpha}{\beta}}$$



$$10)\ \lim_{x\to 0}\frac{tg3x+Sin5x}{2x}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{tg3x+Sin5x}{2x}=\lim_{x\to 0}\frac{8x}{2x}=4$$

$$11)\ \lim_{x\to 0}\left(x\cdot\cos\frac{1}{x}\right)=o$$

$\lim\limits_{x\to 0}(x)=o \Rightarrow$ **x - бм**

$\left|\cos\frac{1}{x}\right|\le 1$ **- ограниченная функция**

$$12)\ \lim_{x\to\infty}\left(x\cdot\sin\frac{1}{x}\right)=[\infty\cdot 0]=\lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}}=\lim_{y\to 0}\frac{\sin y}{y}=1$$

$$\sin\frac{1}{x}\cdot x=\sin\frac{1}{x}:\frac{1}{x}=\frac{\sin\frac{1}{x}}{\frac{1}{x}}\qquad y=\frac{1}{x}\qquad x\to\infty\ \Rightarrow y\to 0$$

$$13)\ \lim_{x\to 0}\frac{x^3}{x-Sinx}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\left(x^3\right)'}{\left(x-Sinx\right)'}=\lim_{x\to 0}\frac{3x^2}{1-Cosx}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{\left(3x^2\right)'}{\left(1-Cosx\right)'}=\lim_{x\to 0}\frac{6x}{Sinx}=6\lim_{x\to 0}\frac{x}{Sinx}=\frac{6}{\lim\limits_{x\to 0}\frac{Sinx}{x}}=6$$

$$\lim_{x\to 0}\frac{Sinx}{x}=1$$

Самостоятельная работа в парах

Вычислить пределы

1) $\lim\limits_{x \to \frac{1}{2}} \dfrac{4x^2 - 1}{2x - 1} =$

2) $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x}{x^2 - x - 6} =$

3) $\lim\limits_{x \to 9} \dfrac{4x^2}{1 + \sqrt{x}} =$

4) $\lim\limits_{x \to 3} \dfrac{x - 3}{x^2 - 6x + 9} =$

5) $\lim\limits_{x \to 0} \dfrac{tg4x}{x} =$

Вычислить пределы ответы

1) $$\lim_{x\to\frac{1}{2}}\frac{4x^2-1}{2x-1}=[\frac{0}{0}]=\lim_{x\to\frac{1}{2}}\frac{(2x-1)(2x+1)}{2x-1}=\lim_{x\to\frac{1}{2}}(2x+1)=2$$

2) $$\lim_{x\to\infty}\frac{3x^2+2x}{x^2-x-6}=\lim_{x\to\infty}\frac{x^2(3+\frac{2}{x})}{x^2(1-\frac{1}{x}-\frac{6}{x^2})}=3$$

$\frac{2}{x}\to 0$, $\frac{1}{x}\to 0$, $\frac{6}{x^2}\to 0$

3) $$\lim_{x\to 9}\frac{4x^2}{1+\sqrt{x}}=\frac{4\cdot 9^2}{1+\sqrt{9}}=81$$

4) $$\lim_{x\to 3}\frac{x-3}{x^2-6x+9}=\lim_{x\to 3}\frac{x-3}{(x-3)^2}=\lim_{x\to 3}\frac{1}{x-3}=\infty$$

5) $$\lim_{x\to 0}\frac{tg4x}{x}=\lim_{x\to 0}\frac{4x}{x}=4$$