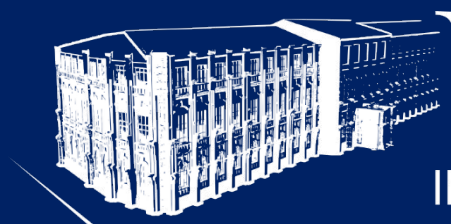

WESTMINSTER

INTERNATIONAL UNIVERSITY IN TASHKENT

An Accredited Institution of the University of Westminster (UK)

LECTURE 3
MEASURES OF DISPERSION

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Lecture outline:

- compute and interpret the (interquartile) range
- compute and interpret the variance
- Compute and interpret the standard deviation
- Interpret confidence interval

Untabulated data.

According to Transfermarkt.ru transfer price of some Uzbek National football team players, are as follows:

Eldor Shomurodov	7 million euros
Otabek Shukurov	3 million euros
Jaloliddin Masharipov	2.8 million euros
Igor Sergeev	1.5 million euros
Odildzhon Khamrobekov	1 million euros
Khozhiakbar Alizhonov	0.8 million euros
Farrukh Sayfiev	0.7 million euros
Dostonbek Tursunov	0.7 million euros
Islomzhon Kobilov	0.6 million euros
Sharof Mukhitdinov	0.55 million euros
Sandzhar Kuvvatov	0.5 million euros

Compute: Range & Interquartile range (IQR).

Range

Range = Max value – Min value

Range = € 7,000,000 - € 500,000 = € 6,500,000

Visually:

€0.5mln €0.55 €0.6 €0.7 €0.7 €0.8 €1.0 €1.5 €2.8 €3.0 **€7.0mln**

€ 6,500,000

Interquartile range

-

$$\mathbf{IQR = Q_3 - Q_1}$$

$\mathbf{Q_1 =} \frac{1}{4}(11+1)th = 3rd$ **= €0.6mln**

$\mathbf{Q_3 =} \frac{3}{4}(11+1)th = 9th$ **= €2.8mln**

Interquartile range

$$IQR = Q_3 - Q_1$$

$IQR = Q_3 - Q_1$ = €2.8mln– €0.6mln= €2.2mln

Visually:

€0.5mln €0.55 **€0.6mln** €0.7 €0.7 €0.8 €1.0 €1.5 **€2.8mln** €3.0 €7.0mln

€2.2mln

Untabulated data

The weekly salaries for the sample of six players of Tampa Bay Buccaneers, are as follows:

$17,000 $20,000 $24,000
$18,000 $23,000; $26,000

Compute: Standard deviation

Standard deviation – Untabulated (1)

1. Compute the mean $\bar{x}$
2. Calculate the difference from the mean $\left(x - \bar{x}\right)$
3. Square these differences $\left(x - \bar{x}\right)^2$
4. Sum the squared differences $\sum \left(x - \bar{x}\right)^2$
5. Average the squared differences to find variance: $s^2 = \frac{\sum \left(x - \bar{x}\right)^2}{n}$
6. Square root variance to find standard deviation: $s = \sqrt{s^2} = \sqrt{\frac{\sum \left(x - \bar{x}\right)^2}{n}}$

Standard deviation – Untabulated (2)

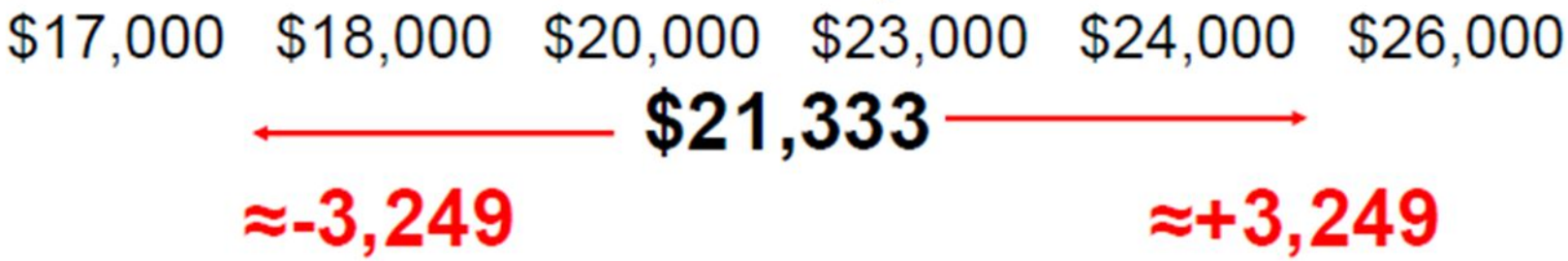
x	$\overline{x}$	$x - \overline{x}$	$(x - \overline{x})^2$
17,000	21,333	- 4,333	18,774,889
18,000	21,333	- 3,333	11,108,889
20,000	21,333	- 1,333	1,776,889
23,000	21,333	1,667	2,778,889
24,000	21,333	2,667	7,112,889
26,000	21,333	4,667	21,780,889
Total			63,333,334

Standard deviation – Untabulated (3)

Empirical rule: For a **normal distribution**:

- Within one standard deviation = **68%** of the data
- Within two standard deviation = **95%** of the data
- Within three standard deviation = Almost all data

Visually:



Tabulated - ungrouped data

The number of Rolton (noodle) sold per day by cashier John in "Next" supermarket in 2014 are shown below:

№ of noodles "Rolton"	10	11	12	13	14	15	16
№ of days	110	70	120	27	18	12	8

Compute: **Range and standard deviation**

Range

Range = Max value – Min value

Range = 16 - 10 = 6

Questions:

Why max and min values are equal to 16 & 10 respectively?

What do call the number of days?

Standard deviation formulae

$$s = \sqrt{s^2} = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$$

Scary formula?

It is hard to solve?

Let's solve it together …

Standard deviation – 1st method (1)

x	f	fx	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
10	110	1100	11.56	– 1.56	2.43	267.3
11	70	770	11.56	– 0.56	0.31	21.7
12	120	1440	11.56	0.44	0.19	22.8
13	27	351	11.56	1.44	2.07	55.89
14	18	252	11.56	2.44	5.95	107.1
15	12	180	11.56	3.44	11.83	141.96
16	8	128	11.56	4.44	19.71	157.68
Total	365	4221				774.43

Standard deviation – 1st method (2)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{4221}{365} = 11.56$$

$$SD = \sqrt{\frac{\sum f\left(x - \bar{x}\right)^2}{\sum f}} = \sqrt{\frac{774.43}{365}} \approx 1.46$$

Standard deviation – 2nd method

$$s = \sqrt{s^2} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\overline{x}\right)^2}$$

Homework:

Try to solve it at home for untabulated data

Tabulated – grouped data

The amount spent on food by 50 people in a particular shop is given in the frequency table below:

Expenditure on food	No. of respondents
£0 – £5	2
£5 – £10	6
£10 – £15	8
£15 – £20	14
£20 – £30	12
£30 – £40	6
£40 – £50	2
Total	**50**

Range

Range = Max value – Min value

Range = £50 - £0 = £50

Questions:

Why max and min values are equal to £50 & £0 respectively?

What do call the number of respondents?

Standard deviation – 2nd method (1)

$$s = \sqrt{s^2} = \sqrt{\frac{\sum f\left(x - \bar{x}\right)^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\bar{x}\right)^2}$$

Expenditure (£) on Food (x)	No. of Respondents (f)	Midpoints	fx	fx^2
0 – 5	2	2.5	5	12.5
5 – 10	6	7.5	45	337.5
10 – 15	8	12.5	100	1,250.0
15 – 20	14	17.5	245	4,287.5
20 – 30	12	25.0	300	7,500.0
30 – 40	6	35.0	210	7,350.0
40 – 50	2	45.0	90	4,050.0
Total	50		995	24,787.5

Standard deviation – 2nd method (2)

$$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f} = \frac{995}{50} = \mathbf{19.9}$$

$$\text{Standard deviation} = s = \sqrt{s^2} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\bar{x}\right)^2} =$$

$$= \sqrt{\frac{24,787.5}{50} - (19.9)^2} = \sqrt{495.75 - 396.01} = \sqrt{99.74} = 9.99 \approx \mathbf{10}$$

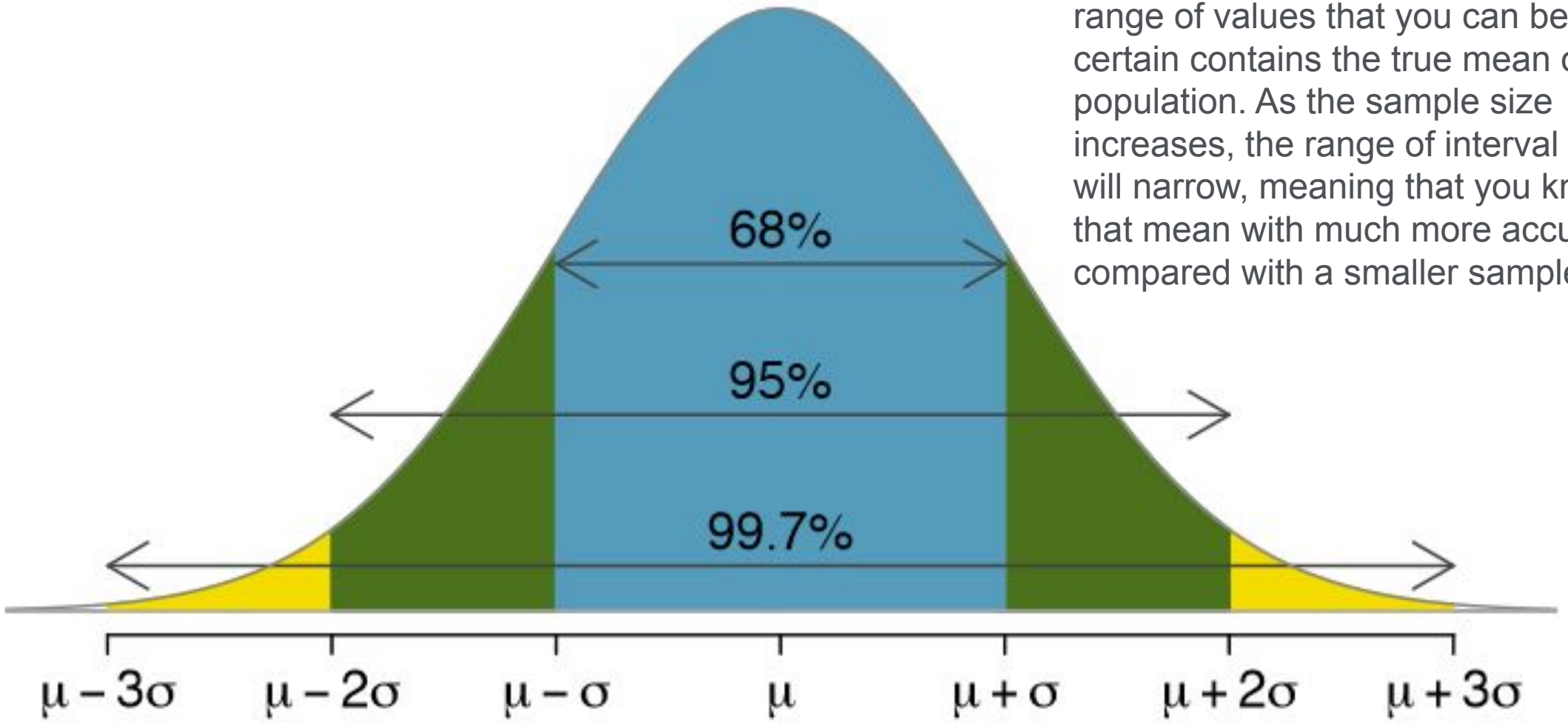
Standard deviation – 1st method

$$s = \sqrt{s^2} = \sqrt{\frac{\sum f\left(x - \bar{x}\right)^2}{\sum f}}$$

Homework:

Try to solve it at home for untabulated data

Confidence interval



The 95% confidence interval is a range of values that you can be 95% certain contains the true mean of the population. As the sample size increases, the range of interval values will narrow, meaning that you know that mean with much more accuracy compared with a smaller sample.

Concluding remarks:

Today, you learnt:

- Use the quantitative methods to find the spread of data
- Calculate the range, interquartile range, and standard deviation

Essential readings:

- Jon Curwin…, “Quantitative methods…”, Ch 6
- Glyn Burton…, “Quantitative methods…”, Ch 2.4
- Richard Thomas, “Quantitative methods…”, Ch 1.8-1.11
- Mik Wisniewski…, “Foundation Quantitative…”, Ch 7
- Clare Morris, “Quantitative Approaches…”, Ch 6
- Louise Swift “Quantitative methods…”, Ch DD2.