ECE 576 – Power System Dynamics and Stability

Lecture 20: Multimachine Simulation

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Announcements

- Read Chapter 7
- Homework 6 is due on Tuesday April 15

Simultaneous Implicit

- The other major solution approach is the simultaneous implicit in which the algebraic and differential equations are solved simultaneously
- This method has the advantage of being numerically stable

Simultaneous Implicit

- Recalling the first lecture, we covered two common implicit integration approaches for solving $\mathbf{x} = \mathbf{f}(\mathbf{x})$
  - Backward Euler

    $$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)$$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\, \mathbf{A}\right]^{-1} \mathbf{x}(t)$$

  - Trapezoidal

    $$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[\mathbf{f}\left(\mathbf{x}(t)\right) + \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)\right]$$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\, \mathbf{A}\right]^{-1}\left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

Nonlinear Trapezoidal

- We can use Newton's method to solve $\mathbf{x} = \mathbf{f}(\mathbf{x})$ with the trapezoidal

$$-\mathbf{x}(t+\Delta t) + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)\right) + \mathbf{f}\left(\mathbf{x}(t)\right)\right) = \mathbf{0}$$

- We are solving for $\mathbf{x}(t+\Delta t)$; $\mathbf{x}(t)$ is known
- The Jacobian matrix is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right) = \frac{\Delta t}{2}\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \end{bmatrix} - \mathbf{I}$$

Right now we are just considering the differential equations; we'll introduce the algebraic equations shortly

Nonlinear Trapezoidal using Newton's Method

- The full solution would be at each time step
  - Set the initial guess for **x**(t+Δt) as **x**(t), and initialize the iteration counter k = 0
  - Determine the mismatch at each iteration k as

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) \triangleq -\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t)\right)\right)$$

  - Determine the Jacobian matrix
  - Solve

$$\mathbf{x}(t+\Delta t)^{(k+1)} = \mathbf{x}(t+\Delta t)^{(k)} - \left[\mathbf{J}(\mathbf{x}(t+\Delta t)^{(k)}\right]^{-1} \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right)$$

  - Iterate until done

Infinite Bus GENCLS Implicit Solution

- Assume a solid three phase fault is applied at the generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear}, resulting in the post-fault system being identical to the pre-fault system
  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}(1-0)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Infinite Bus GENCLS Implicit Solution

- The initial conditions are

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Let $\Delta t = 0.02$ seconds
- During the fault the Jacobian is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right) = \frac{0.02}{2}\begin{bmatrix} 0 & \omega_s \\ 0 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}$$

- Set the initial guess for **x**(0.02) as **x**(0), and

$$\mathbf{f}\left(\mathbf{x}(0)\right) = \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Then calculate the initial mismatch

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) = -\mathbf{x}(0.02)^{(0)} + \mathbf{x}(0) + \frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.02)^{(0)}\right) + \mathbf{f}\left(\mathbf{x}(0)\right)\right)$$

- With $\mathbf{x}(0.02)^{(0)} = \mathbf{x}(0)$ this becomes

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) = -\begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2}\left(\begin{bmatrix} 0 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix}$$

- Then

$$\mathbf{x}(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix} = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Repeating for the next iteration

$$\mathbf{f}\left(\mathbf{x}(0.02)^{(1)}\right) = \begin{bmatrix} 1.259 \\ 0.1667 \end{bmatrix}$$

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(1)}\right) = -\begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2}\left(\begin{bmatrix} 1.259 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

- Hence we have converged with $\mathbf{x}(0.02) = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$

Infinite Bus GENCLS Implicit Solution

- Iteration continues until t = T^{clear}, assumed to be 0.1 seconds in this example

$$\mathbf{x}(0.10) = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix}$$

- At this point, when the fault is self-cleared, the equations change, requiring a re-evaluation of $\mathbf{f}(\mathbf{x}(T^{clear}))$

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

$$\mathbf{f}\left(\mathbf{x}\left(0.1^{+}\right)\right) = \begin{bmatrix} 6.30 \\ -0.1078 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- With the change in f(x) the Jacobian also changes

$$\mathbf{J}\left(\mathbf{x}(0.12^{(0)})\right) = \frac{0.02}{2}\begin{bmatrix} 0 & \omega_s \\ -0.305 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}$$

- Iteration for **x**(0.12) is as before, except using the new function and new Jacobian

$$\mathbf{h}\left(\mathbf{x}(0.12)^{(0)}\right) \triangleq -\mathbf{x}(0.12)^{(0)} + \mathbf{x}(0.01) + \frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.12)^{(0)}\right) + \mathbf{f}\left(\mathbf{x}(0.10^{+})\right)\right)$$

$$\mathbf{x}(0.12)^{(1)} = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0.1257 \\ -0.00216 \end{bmatrix} = \begin{bmatrix} 0.848 \\ 0.0142 \end{bmatrix}$$

This also converges quickly, with one or two iterations

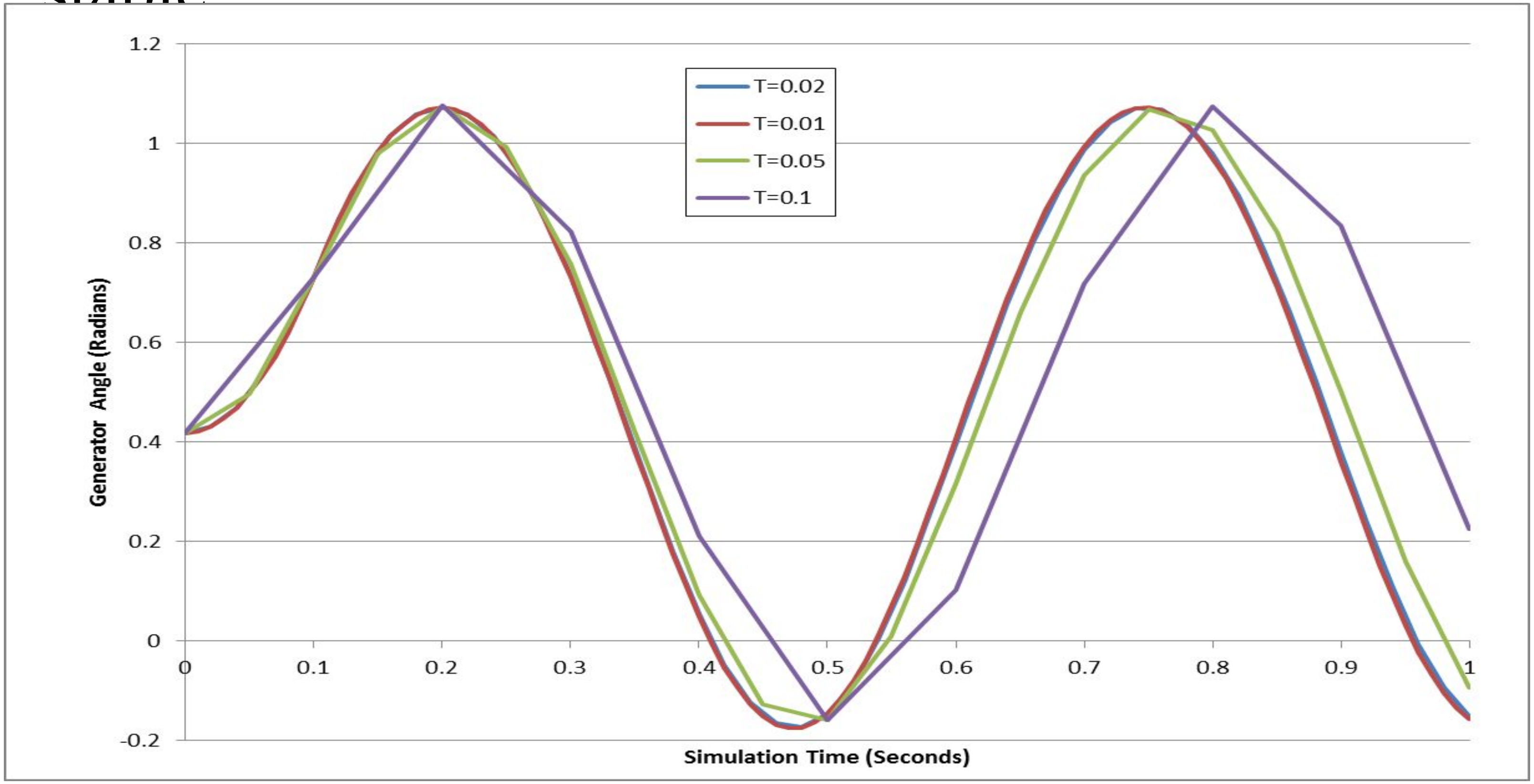
Computational Considerations

- As presented for a large system most of the computation is associated with updating and factoring the Jacobian. But the Jacobian actually changes little and hence seldom needs to be rebuilt/factored
- Rather than using **x**(t) as the initial guess for **x**(t+Δt), prediction can be used when previous values are available

$$\mathbf{x}(t+\Delta t)^{(0)} = \mathbf{x}(t) + \left(\mathbf{x}(t) - \mathbf{x}(t-\Delta t)\right)$$

Two Bus Results

- The below graph shows the generator angle for varying values of Δt; recall the implicit method is numerically stable



Adding the Algebraic Constraints

- Since the classical model can be formulated with all the values on the network reference frame, initially we just need to add the network equations
- We'll again formulate the network equations using the form $\mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{Y}\,\mathbf{V}$ or $\mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{0}$
- As before the complex equations will be expressed using two real equations, with voltages and currents expressed in rectangular coordinates

Adding the Algebraic Constraints

- The network equations are as before

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad \mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \sum_{k=1}^{n} \left(G_{1k} V_{Dk} - B_{1k} V_{QK} \right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{2k} V_{Dk} - B_{2k} V_{QK} \right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^{n} \left(G_{nk} V_{Dk} - B_{nk} V_{QK} \right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{nk} V_{Qk} + B_{nk} V_{DK} \right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0 \end{bmatrix}$$

Classical Model Coupling of x and y

- In the simultaneous implicit method **x** and **y** are determined simultaneously; hence in the Jacobian we need to determine the dependence of the network equations on **x**, and the state equations on **y**
- With the classical model the Norton current depends on **x** as

$$\bar{I}_{Ni} = \frac{E_i \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad G_i + jB_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$

$$\bar{I}_{Ni} = I_{DNi} + jI_{QNi} = E'_i\left(\cos\delta_i + j\sin\delta_i\right)\left(G_i + jB_i\right)$$

$$E_{Di} + jE_{Qi} = E'_i\left(\cos\delta_i + j\sin\delta_i\right)$$

$$I_{DNi} = E_{Di}G_i - E_{Qi}B_i$$

$$I_{QNi} = E_{Di}B_i + E_{Qi}G_i$$

Classical Model Coupling of x and y

- The in the state equations the coupling with **y** is recognized by noting

$$\mathrm{P}_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$$I_{Di} + jI_{Qi} = \left(\left(E_{Di} - V_{Di}\right) + j\left(E_{Qi} - V_{Qi}\right)\right)\left(G_i + jB_i\right)$$

$$I_{Di} = \left(E_{Di} - V_{Di}\right)G_i - \left(E_{Qi} - V_{Qi}\right)B_i$$

$$I_{Qi} = \left(E_{Di} - V_{Di}\right)B_i + \left(E_{Qi} - V_{Qi}\right)G_i$$

$$\mathrm{P}_{Ei} = E_{Di}\left(\left(E_{Di} - V_{Di}\right)G_i - \left(E_{Qi} - V_{Qi}\right)B_i\right) + E_{Qi}\left(\left(E_{Di} - V_{Di}\right)B_i + \left(E_{Qi} - V_{Qi}\right)G_i\right)$$

$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

Variables and Mismatch Equations

- In solving the Newton algorithm the variables now include **x** and **y** (recalling that here **y** is just the vector of the real and imaginary bus voltages
- The mismatch equations now include the state integration equations

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) =$$

$$-\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t), \mathbf{y}(t)\right)\right)$$

- And the algebraic equations

$$\mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right)$$

Jacobian Matrix

- Since the **h**(**x**,**y**) and **g**(**x**,**y**) are coupled, the Jacobian is

$$J\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)$$

$$=\begin{bmatrix} \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \\ \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \end{bmatrix}$$

  - With the classical model the coupling is the Norton current at bus i depends on δ_i (i.e., **x**) and the electrical power (P_{Ei}) in the swing equation depends on V_{Di} and V_{Qi} (i.e., **y**)

Jacobian Matrix Entries

- The dependence of the Norton current injections on δ is

$$I_{DNi} = E_i' \cos\delta_i G_i - E_i' \sin\delta_i B_i$$

$$I_{QNi} = E_i' \cos\delta_i B_i + E_i' \sin\delta_i G_i$$

$$\frac{\partial I_{DNi}}{\partial \delta_i} = -E_i' \sin\delta_i G_i - E_i' \cos\delta_i B_i$$

$$\frac{\partial I_{QNi}}{\partial \delta_i} = -E_i' \sin\delta_i B_i + E_i' \cos\delta_i G_i$$

  - In the Jacobian the sign is flipped because we defined

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x}, \mathbf{y})$$

Jacobian Matrix Entries

- The dependence of the swing equation on the generator terminal voltage is

$$\dot{\delta}_i = \Delta\omega_{i.pu}\omega_s$$

$$\Delta\dot{\omega}_{i,pu} = \frac{1}{2H_i}\left(P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

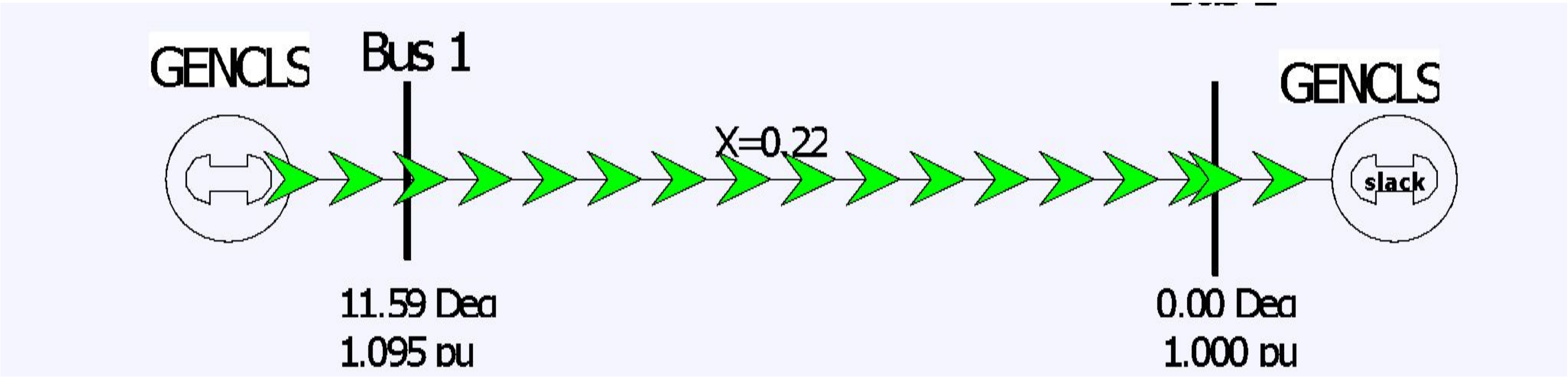
$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Di}} = \frac{1}{2H_i}\left(E_{Di}G_i + E_{Qi}B_i\right)$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Qi}} = \frac{1}{2H_i}\left(E_{Qi}G_i - E_{Di}B_i\right)$$

Two Bus, Two Gen GENCLS Example

- We'll reconsider the two bus, two generator case from Lecture 18; fault at Bus 1, cleared after 0.06 seconds
  - Initial conditions and $\mathbf{Y}_{bus}$ are as covered in Lecture 18



PowerWorld Case B2_CLS_2Gen

Two Bus, Two Gen GENCLS Example

- Initial terminal voltages are

$$V_{D1} + jV_{Q1} = 1.0726 + j0.22, \quad V_{D2} + jV_{Q2} = 1.0$$

$$\bar{E}_1 = 1.281\angle 23.95°, \quad \bar{E}_2 = 0.955\angle -12.08$$

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

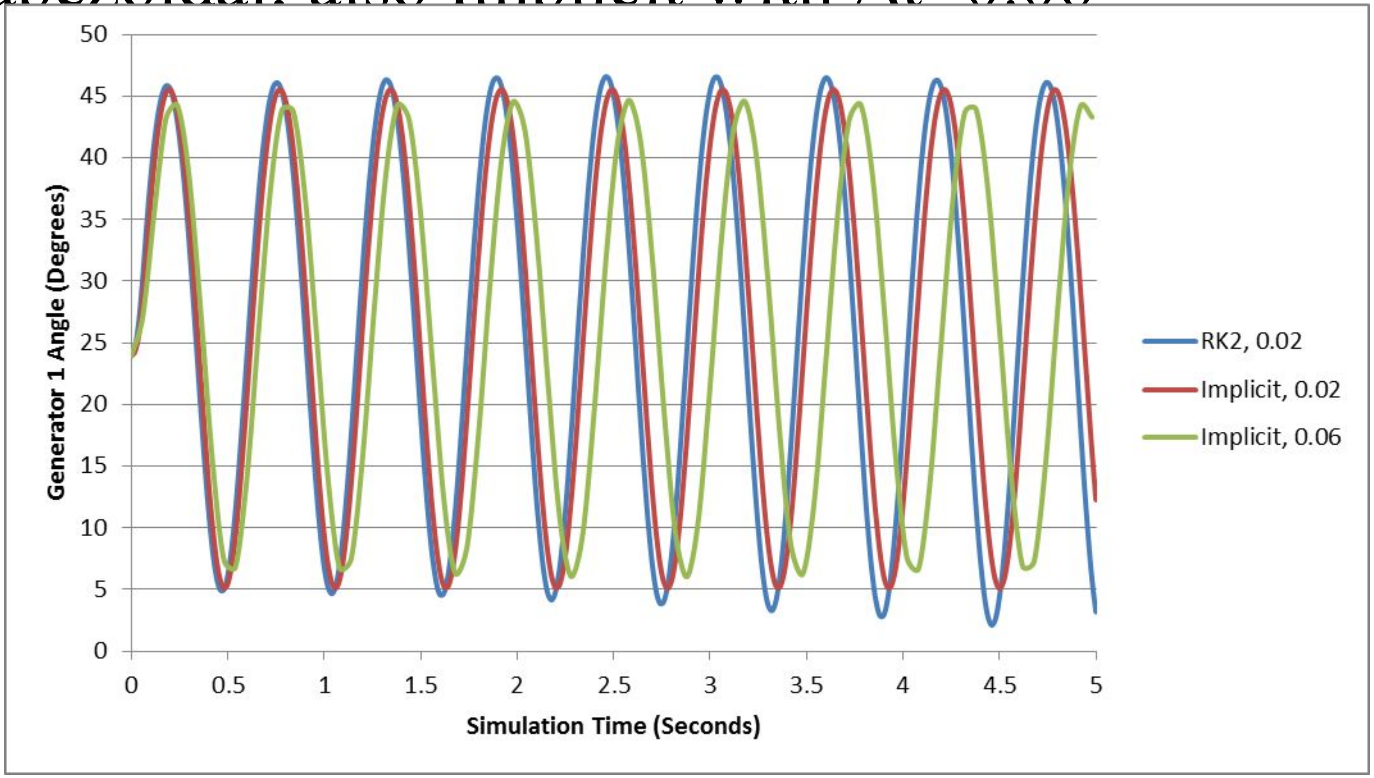
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.333} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

Two Bus, Two Gen Initial Jacobian

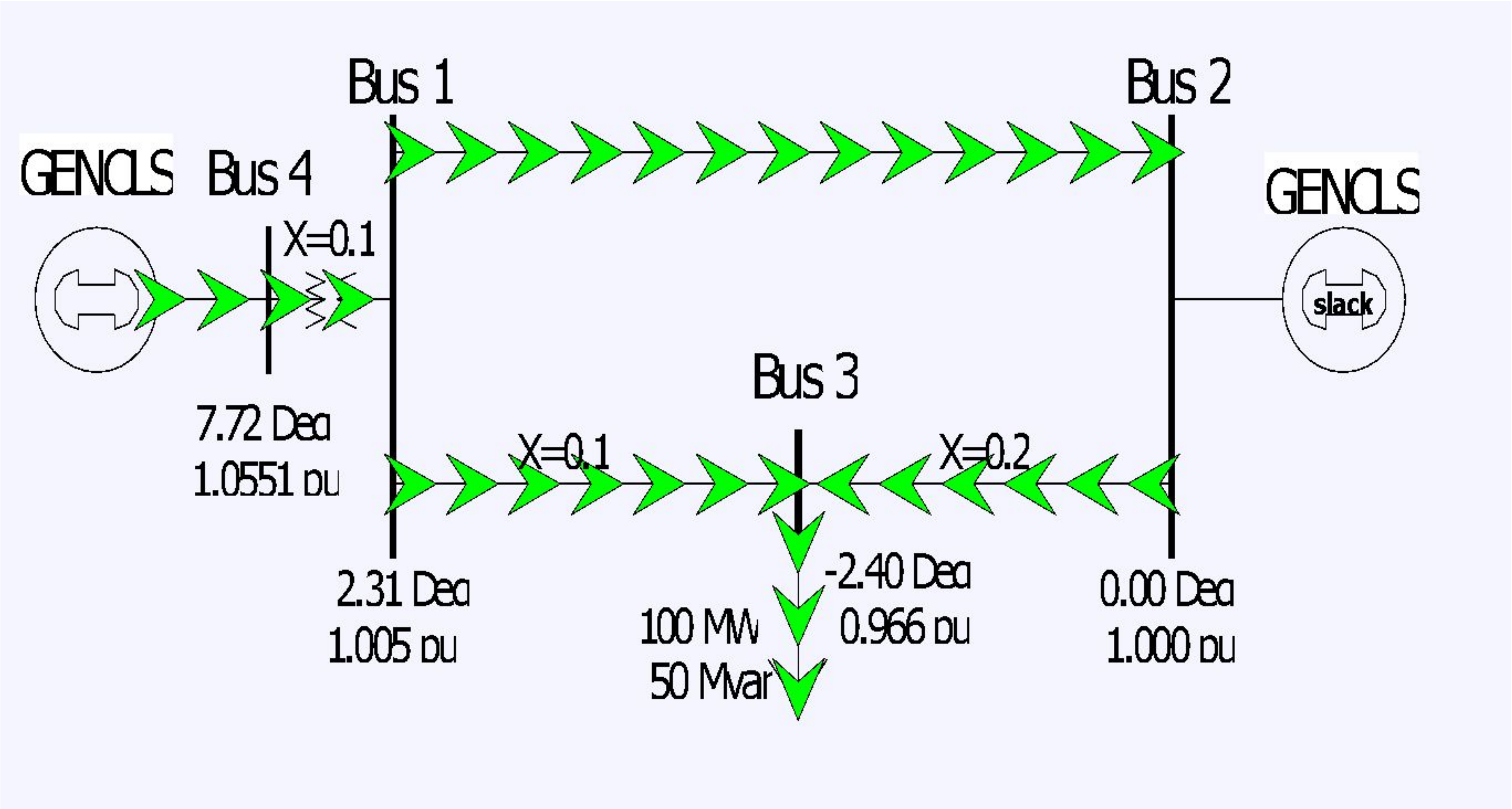
$$\begin{bmatrix} & \delta_1 & \Delta\omega_1 & \delta_2 & \Delta\omega_1 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} \\ \dot{\delta}_1 & -1 & 3.77 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta\dot{\omega}_1 & -0.0076 & -1 & 0 & 0 & -0.0029 & 0.0065 & 0 & 0 \\ \dot{\delta}_2 & 0 & 0 & -1 & 3.77 & 0 & 0 & 0 & 0 \\ \Delta\dot{\omega}_2 & 0 & 0 & -0.0039 & -1 & 0 & 0 & 0.0008 & 0.0039 \\ I_{D1} & -3.90 & 0 & 0 & 0 & 0 & 7.879 & 0 & -4.545 \\ I_{Q1} & -1.73 & 0 & 0 & 0 & -7.879 & 0 & 4.545 & 0 \\ I_{D2} & 0 & 0 & -4.67 & 0 & 0 & -4.545 & 0 & 9.545 \\ I_{Q2} & 0 & 0 & 1.00 & 0 & 4.545 & 0 & -9.545 & 0 \end{bmatrix}$$

Results Comparison

- The below graph compares the angle for the generator at bus 1 using Δt=0.02 between RK2 and the Implicit Trapezoidal; also Implicit with Δt=0.06



Four Bus Comparison



Four Bus Comparison

Fault at Bus 3 for 0.12 seconds; self-cleared