Interference:
An Information Theoretic View

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Thanks: Changho Suh.

Context

- Two central phenomena in wireless communications:
  - Fading
  - Interference
- Much progress on information theory of fading channels in the past 15 years
- Led to important communication techniques:
  - MIMO
  - Opportunistic communication
- Already implemented in many wireless systems.

Interference

- These techniques improve point-to-point and single cell (AP) performance.
- But performance in wireless systems are often limited by interference between multiple links.
- Two basic approaches:
  - orthogonalize into different bands
  - full sharing of spectrum but treating interference as noise
- What does information theory have to say about the optimal thing to do?

State-of-the-Art

- The capacity of even the simplest two-user interference channel (IC) is open for 30 years.
- But significant progress has been made in the past few years through approximation results.
- Some new ideas:
  - generalized degrees of freedom
  - deterministic modeling
  - interference alignment.
- Goal of the tutorial is to explain these ideas.

Outline

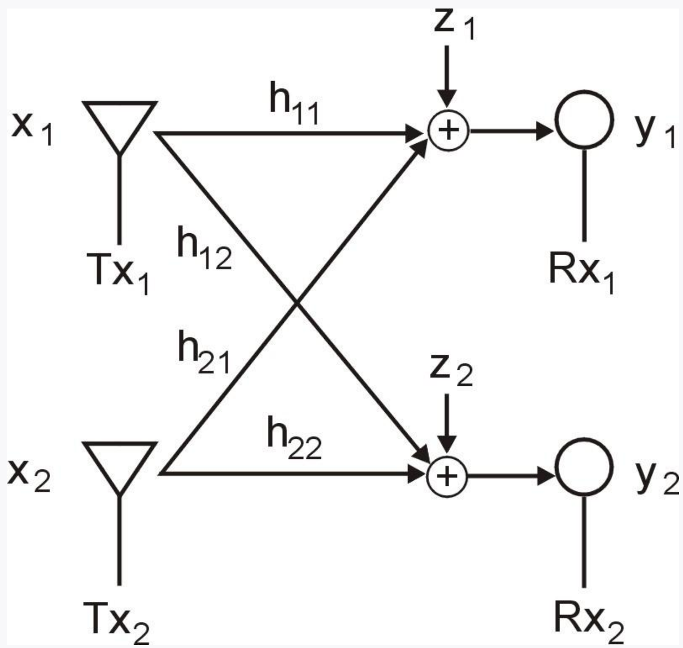
- Part 1: two-user Gaussian IC.
- Part 2: Resource-sharing view and role of feedback and cooperation.
- Part 3: Multiple interferers and interference alignment.

Part I: 2-User Gaussian IC

Two-User Gaussian Interference Channel

message m_1 — want m_1

message m_2 — want m_2



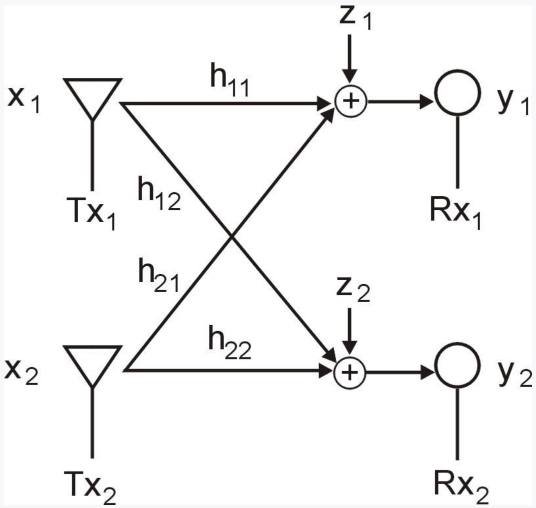
- Characterized by 4 parameters:
  - Signal-to-noise ratios SNR_1, SNR_2 at Rx 1 and 2.
  - Interference-to-noise ratios $INR_{2\text{->}1}$, $INR_{1\text{->}2}$ at Rx 1 and 2.

Related Results

- If receivers can cooperate, this is a multiple access channel. Capacity is known. (Ahlswede 71, Liao 72)

- If transmitters can cooperate , this is a MIMO broadcast channel. Capacity recently found.

  (Weingarten et al 05)



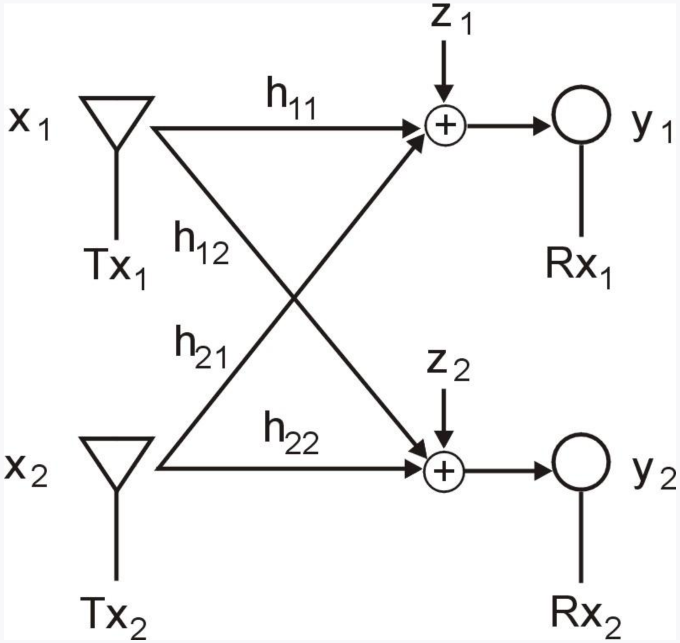
- When there is no cooperation of all, it's the interference channel. Open problem for 30 years.

State-of-the-Art in 2006

- If $INR_{1\text{->}2} > SNR_1$ and $INR_{2\text{->}1} > SNR_2$, then capacity region C_{int} is known (strong interference, Han-Kobayashi 1981, Sato 81)
- Capacity is unknown for any other parameter ranges.
- Best known achievable region is due to Han-Kobayashi (1981).
- Hard to compute explicitly.
- Unclear if it is optimal or even how far from capacity.
- Some outer bounds exist but unclear how tight (Sato 78, Costa 85, Kramer 04).

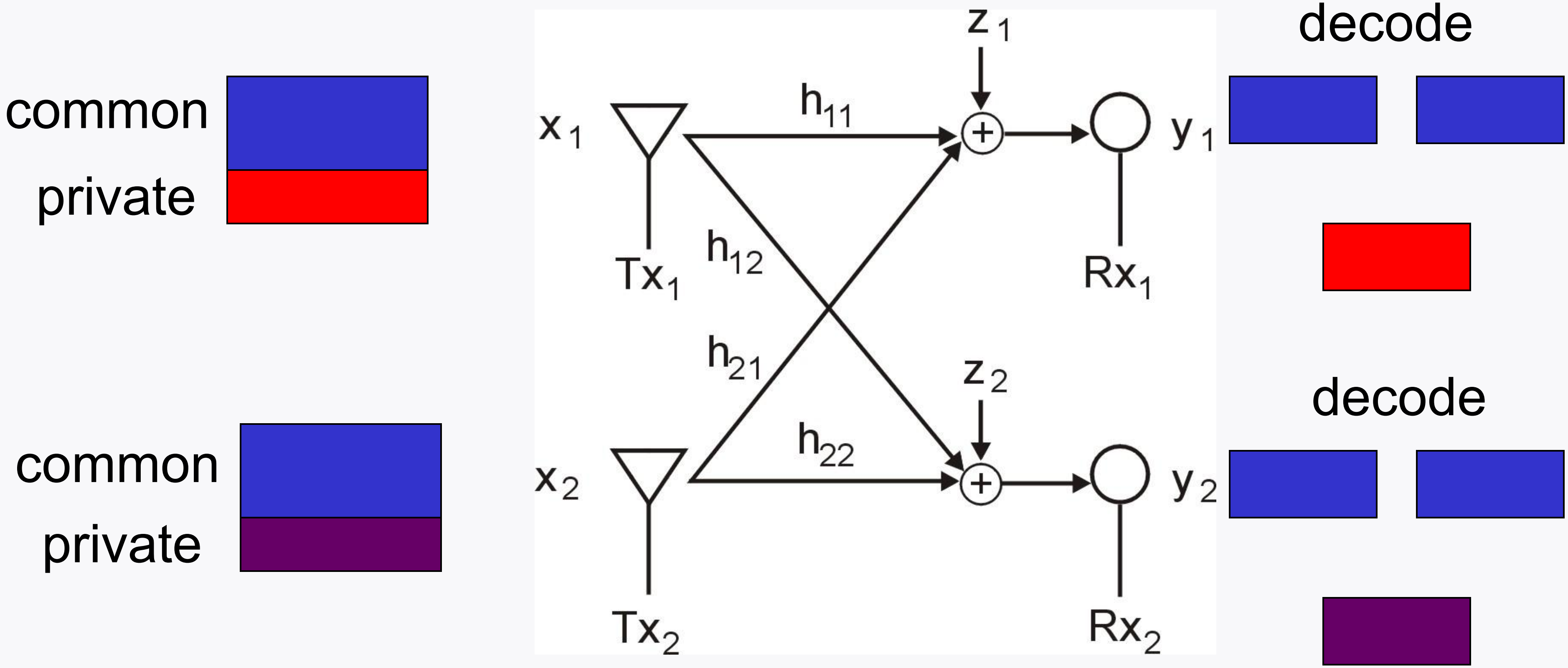
Review: Strong Interference Capacity

- $INR_{1\text{->}2} > SNR_1$, $INR_{2\text{->}1} > SNR_2$



- Key idea: in any achievable scheme, each user must be able to decode the other user's message.
- Information sent from each transmitter must be common information, decodable by all.
- The interference channel capacity region is the intersection of the two MAC regions, one at each receiver.

Han-Kobayashi Achievable Scheme



- Problems of computing the HK region:
  - optimal auxillary r.v.'s unknown
  - time-sharing over many choices of auxillary r.v,'s may be required.

Interference-Limited Regime

- At low SNR, links are noise-limited and interference plays little role.

- At high SNR and high INR, links are interference-limited and interference plays a central role.

- Classical measure of performance in the high SNR regime is the degree of freedom.

Baselines (Symmetric Channel)

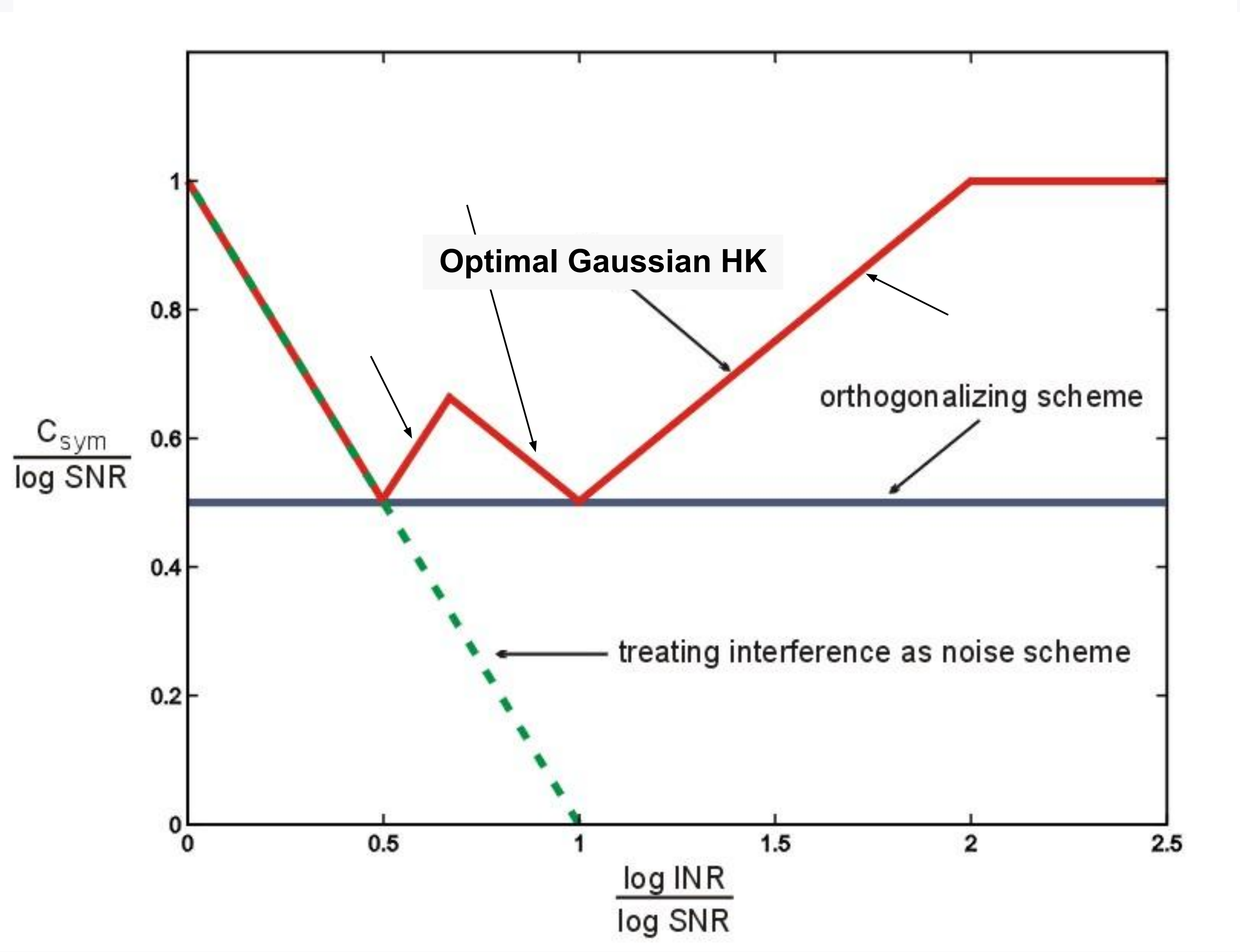
- Point-to-point capacity:
- Achievable rate by orthogonalizing:
- Achievable rate by treating interference as noise:

Generalized Degrees of Freedom

- Let both SNR and INR to grow, but fixing the ratio:

- Treating interference as noise:

Dof plot

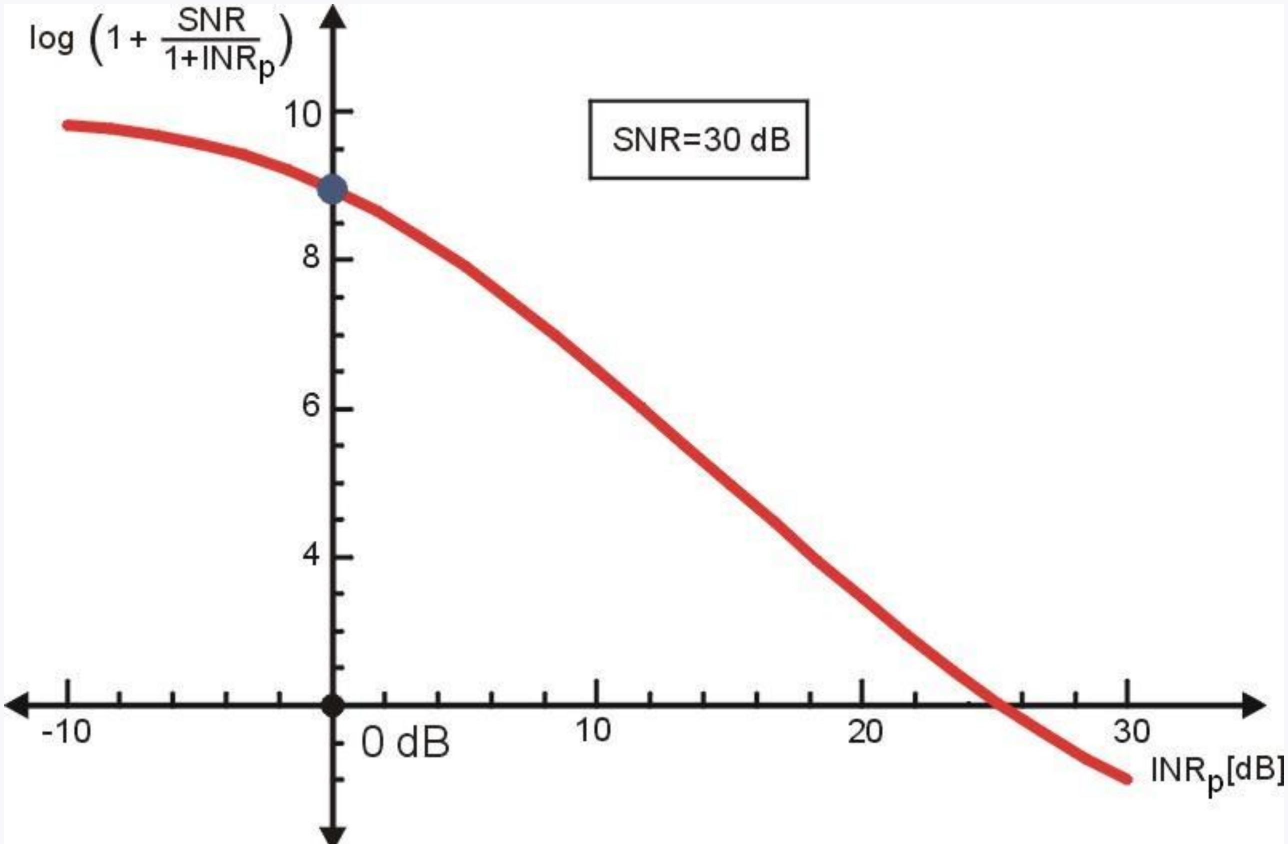


Optimal Gaussian HK
orthogonalizing scheme
treating interference as noise scheme
$\frac{C_{sym}}{\log SNR}$
$\frac{\log INR}{\log SNR}$
1
0.8
0.6
0.4
0.2
0
0.5
1.5
2
2.5

Dof-Optimal Han-Kobayashi

- Only a single split: no time-sharing.
- Private power set so that interference is received at noise level at the other receiver.

Why set INR_p = 0 dB?

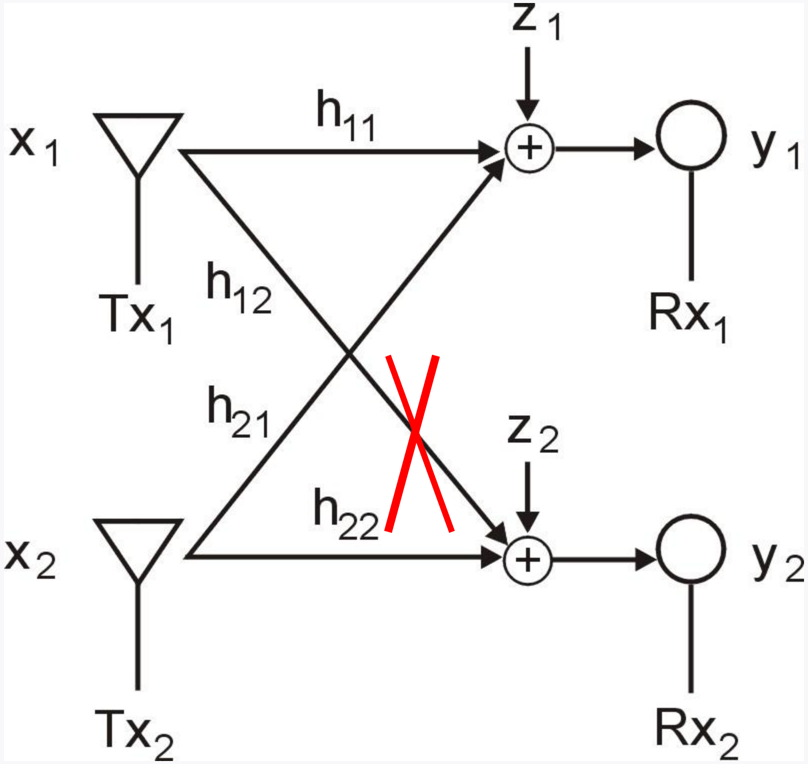


- This is a sweet spot where the damage to the other link is small but can get a high rate in own link since SNR > INR.

Can we do Better?

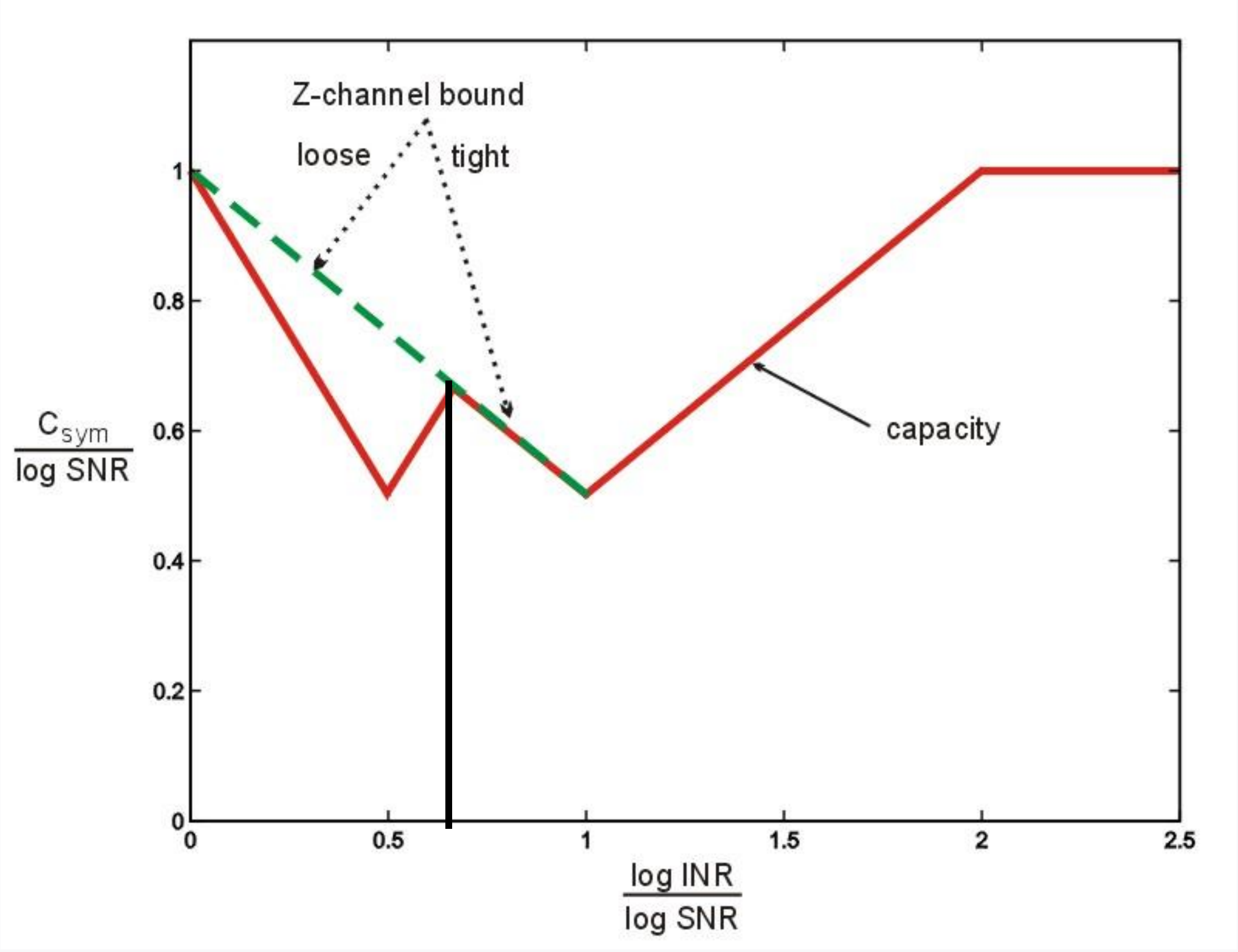
- We identified the Gaussian HK scheme that achieves optimal gdof.
- But can one do better by using non-Gaussian inputs or a scheme other than HK?
- Answer turns out to be no.
- The gdof achieved by the simple HK scheme is the gdof of the interference channel.
- To prove this, we need outer bounds.

Upper Bound: Z-Channel



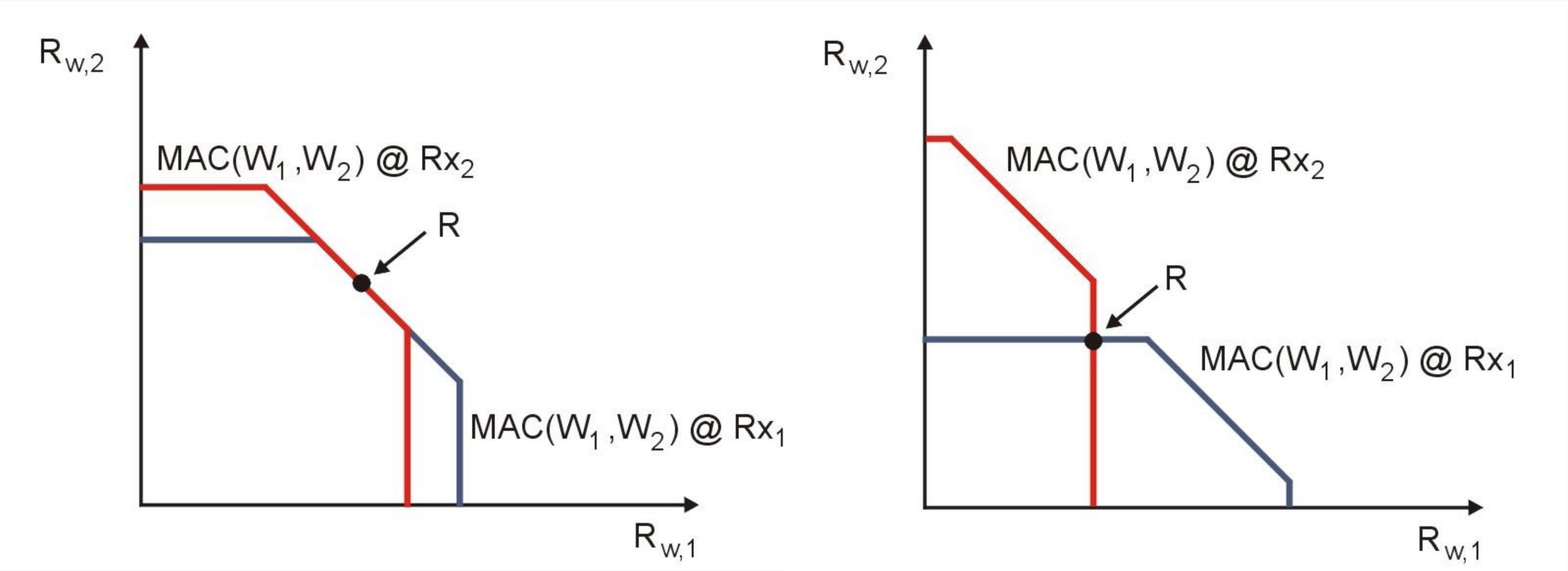
- Equivalently, x_1 given to Rx 2 as side information.

How Good is this Bound?



What's going on?

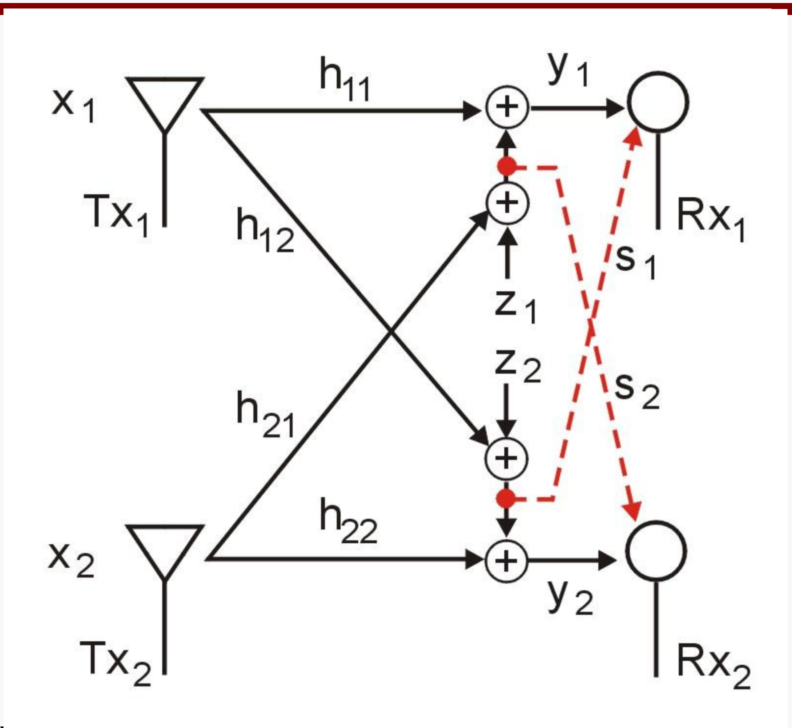
Scheme has 2 distinct regimes of operation:



Z-channel bound is tight.

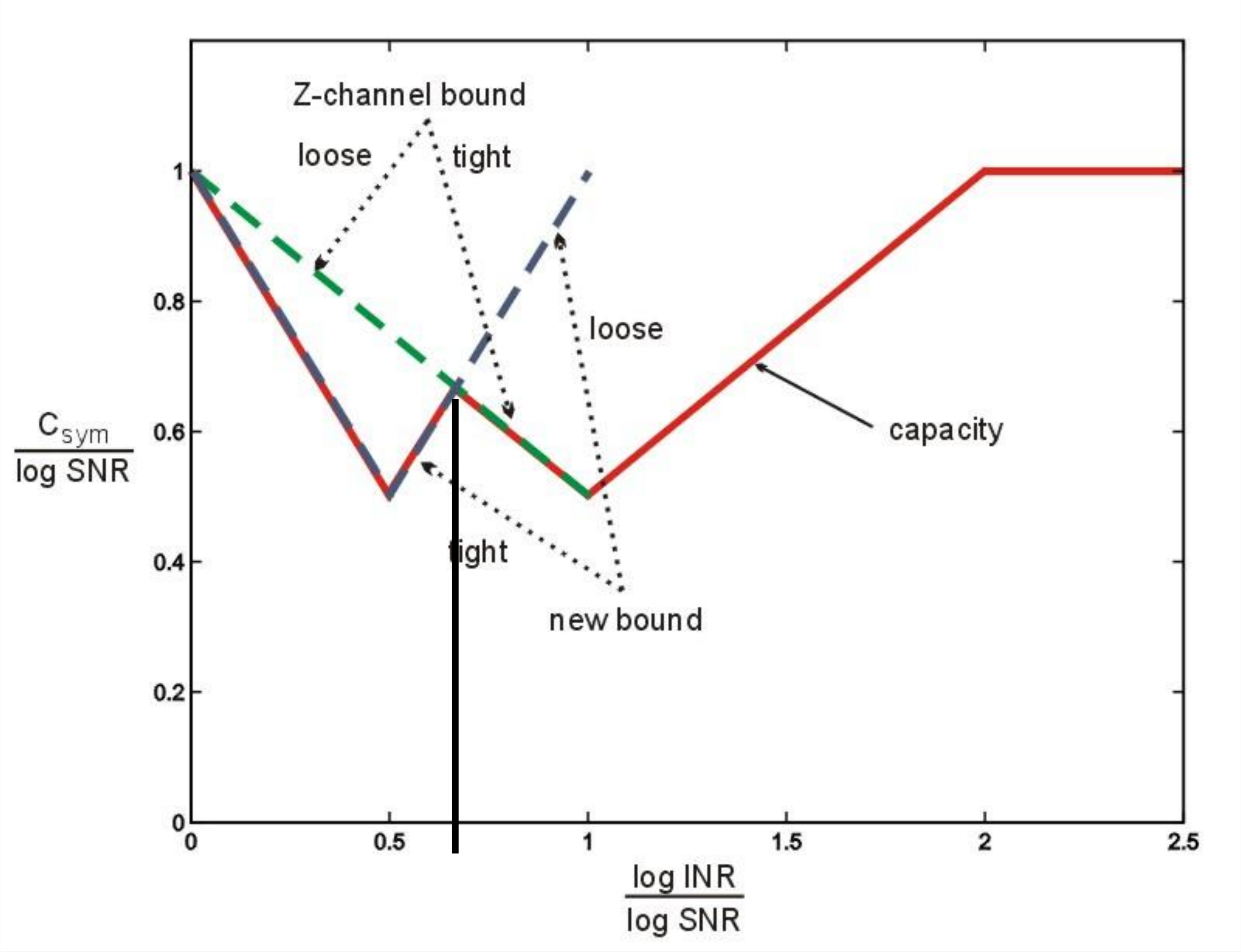
Z-channel bound is not tight.

New Upper Bound



- Genie only allows to give away the common information of user i to receiver i.
- Results in a new interference channel.
- Capacity of this channel can be explicitly computed!

New Upper Bound + Z-Channel Bound is Tight



Back from Infinity

In fact, the simple HK scheme can achieve within 1 bit/s/Hz of capacity for all values of channel parameters:

For any in C_{int}, this scheme can achieve rates

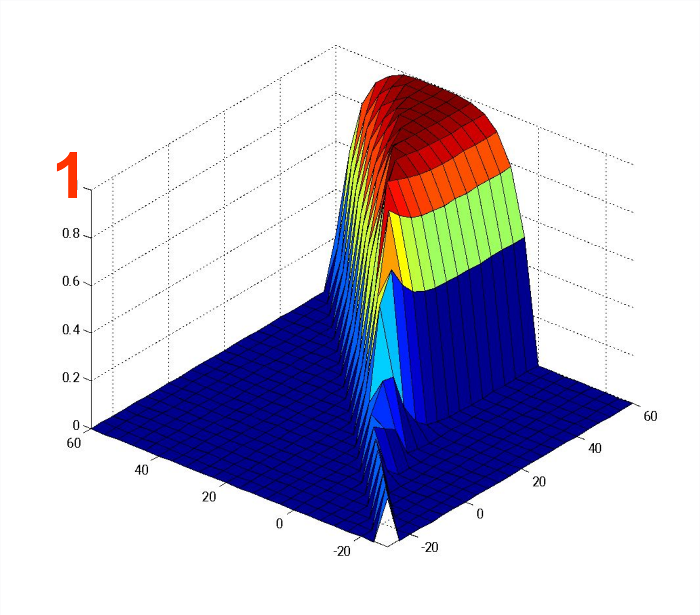
(Etkin, T. & Wang 06)

Symmetric Weak Interference

The scheme achieves a symmetric rate per user:

The symmetric capacity is upper bounded by:

The gap is at most one bit for all values of SNR and INR.



From 1-Bit to 0-Bit

The new upper bound can further be sharpened to get exact results in the low-interference regime (α < 1/3).

(Shang,Kramer,Chen 07,
Annaprueddy & Veeravalli08,
Motahari&Khandani07)

From Low-Noise to No-Noise

- The 1-bit result was obtained by first analyzing the dof of the Gaussian interference channel in the low-noise regime .
- Turns out there is a deterministic interference channel which captures exactly the behavior of the interference-limited Gaussian channel.
- Identifying this underlying deterministic structure allows us to generalize the approach.

Part 2: Resource, Feedback and Cooperation

Basic Questions

1) How to abstract a higher view of the 2-user IC result?

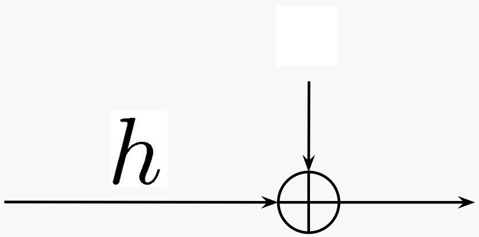
2) In particular: how to quantify the resource being shared?

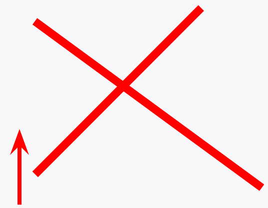
The key is deterministic modeling of the IC.

Point-to-Point Communication: An Abstraction

Transmit a real number

$x = 0.b_1 b_2 b_3 b_4 b_5$





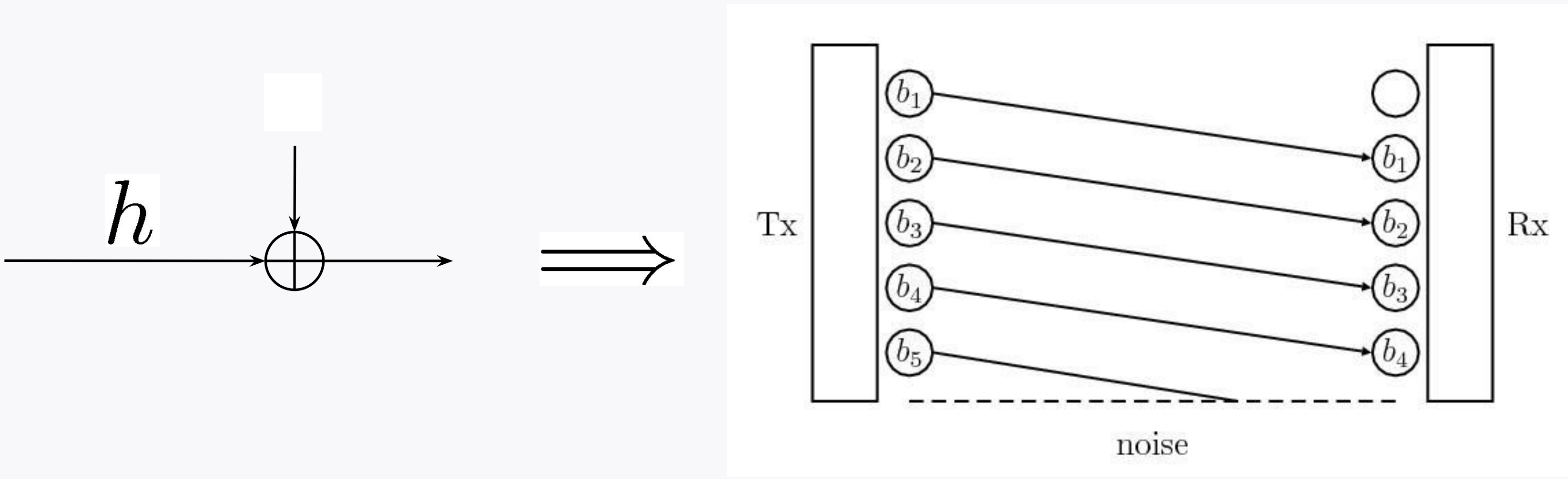
Least significant bits are truncated at noise level.

$$n \leftrightarrow \log_2 \mathsf{SNR}$$

Matches approx:

$$C_{\mathsf{awgn}}(\mathsf{SNR}) = \log(1 + \mathsf{SNR})$$

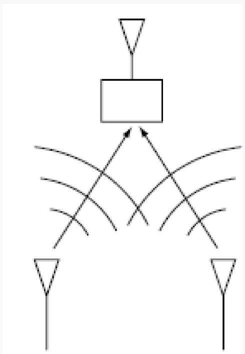
A Deterministic Model



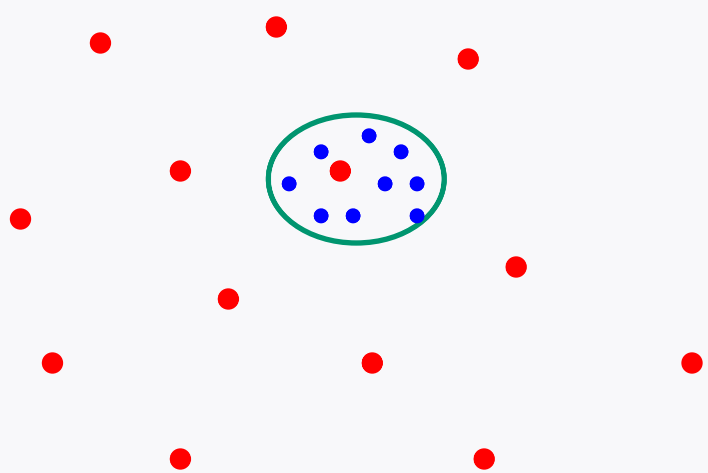
(Avestimehr,Diggavi & T. 07)

Superposition

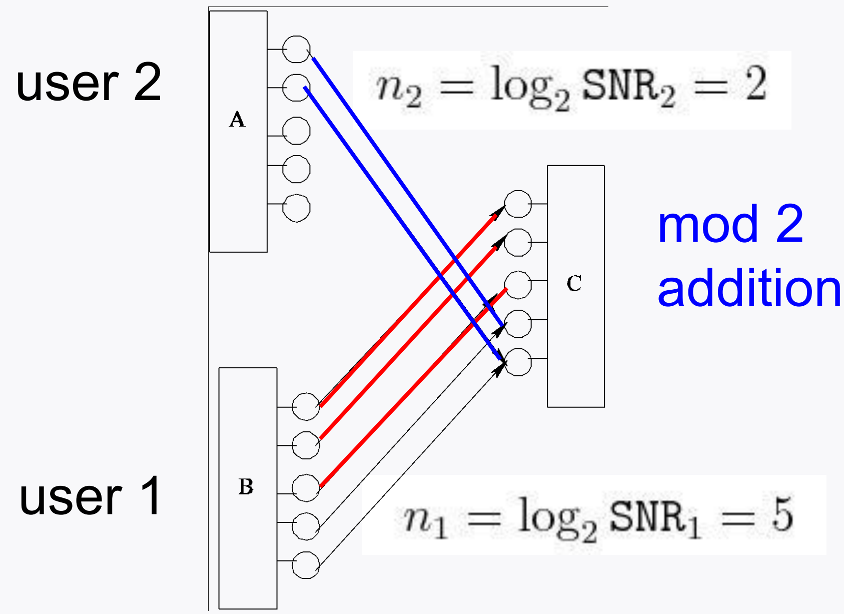
Gaussian



user 1 sends cloud centers, user 2 sends clouds.

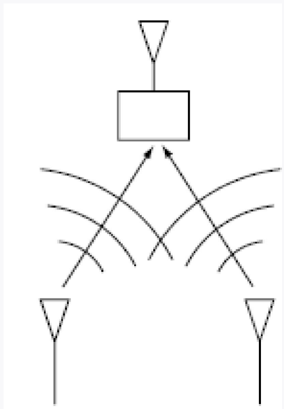


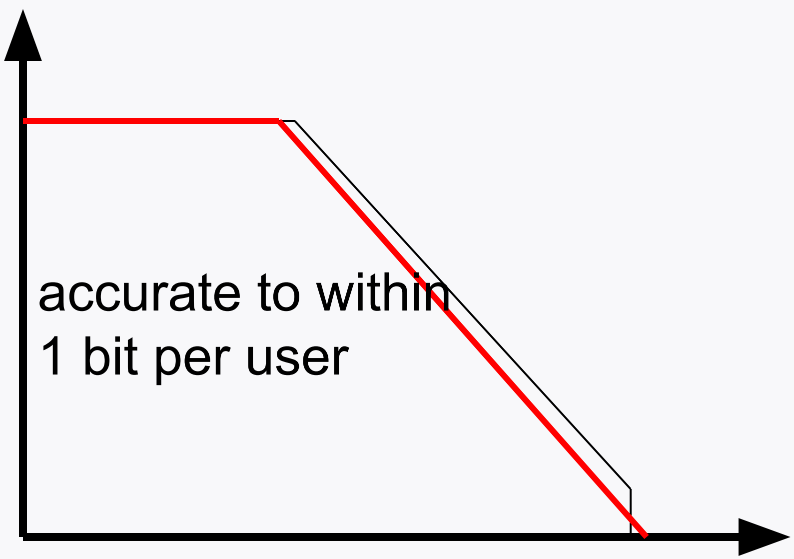
Deterministic



Comparing Multiple Access Capacity Regions

Gaussian

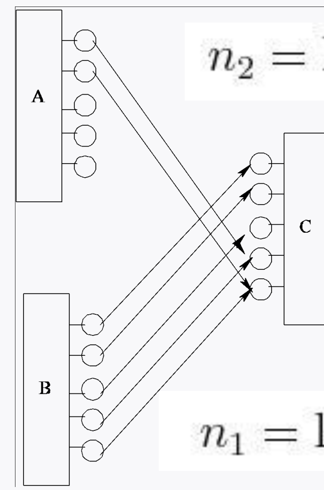




Deterministic

user 2

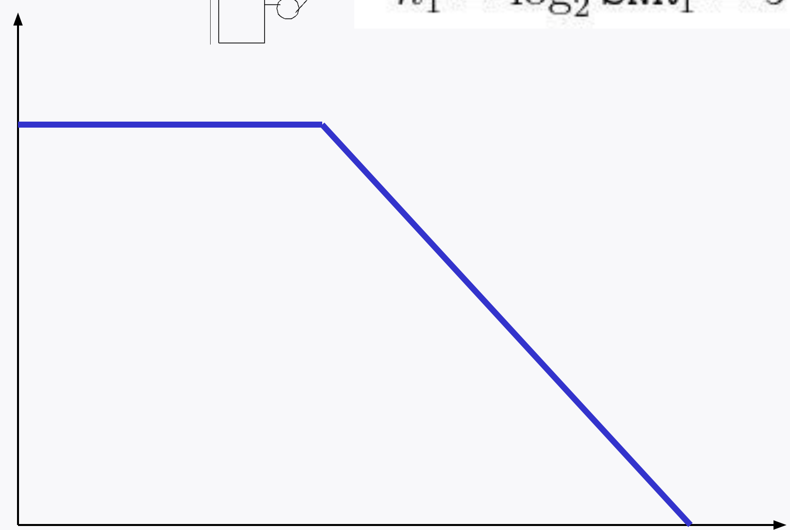
user 1



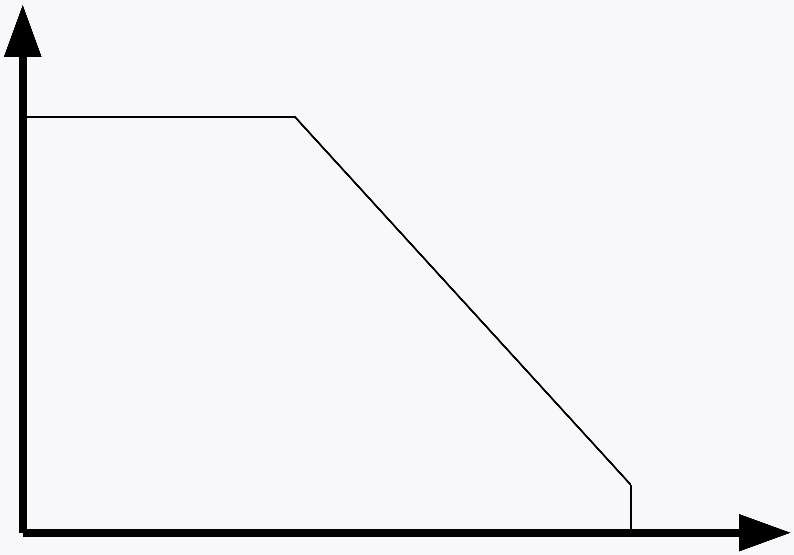
$$n_2 = \log_2 \mathrm{SNR}_2 = 2$$

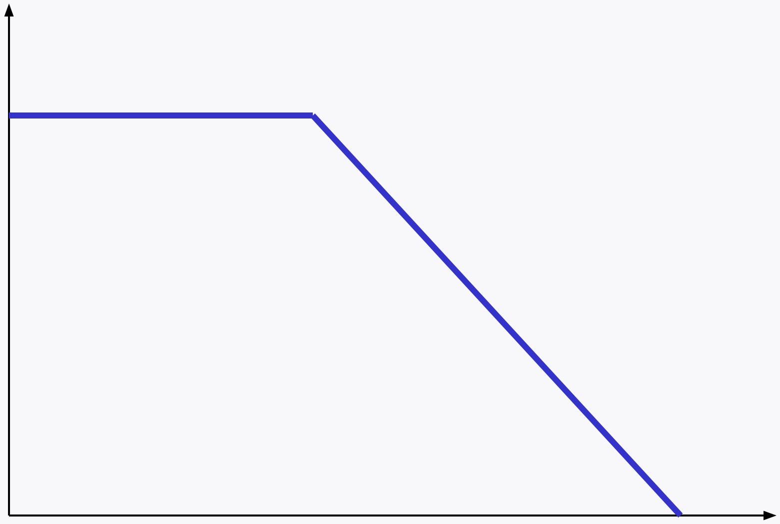
mod 2 addition

$$n_1 = \log_2 \mathrm{SNR}_1 = 5$$



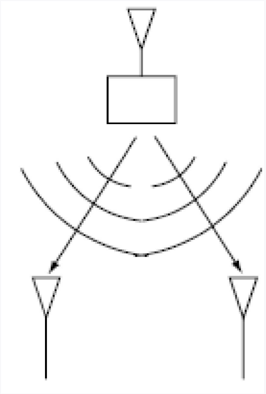
Generalized Degrees of Freedom



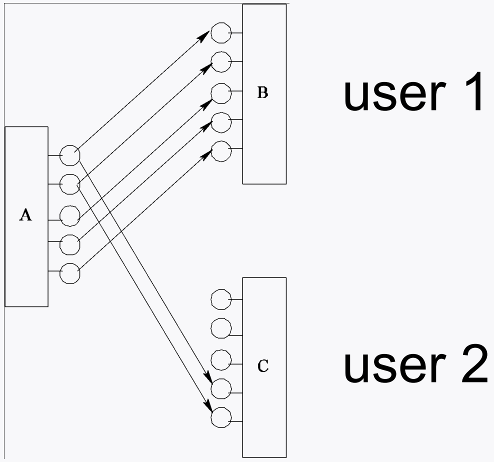


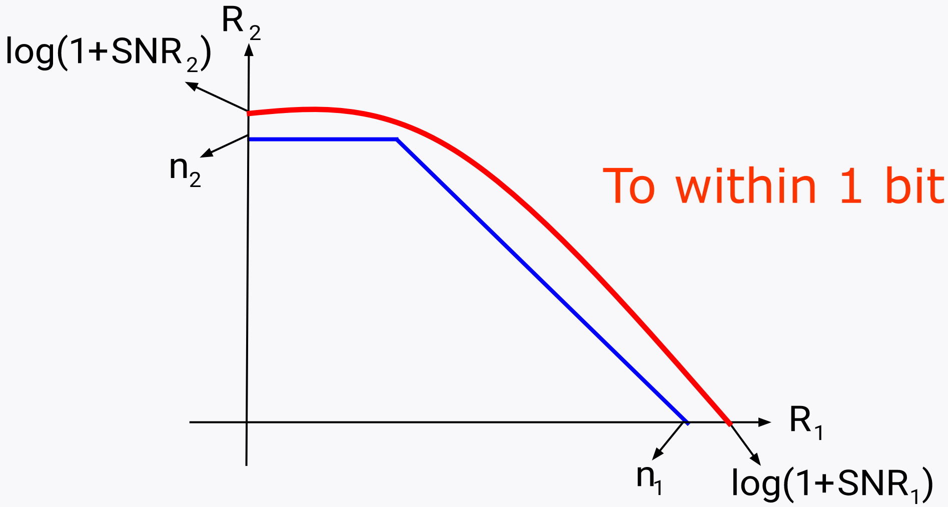
Broadcast

Gaussian



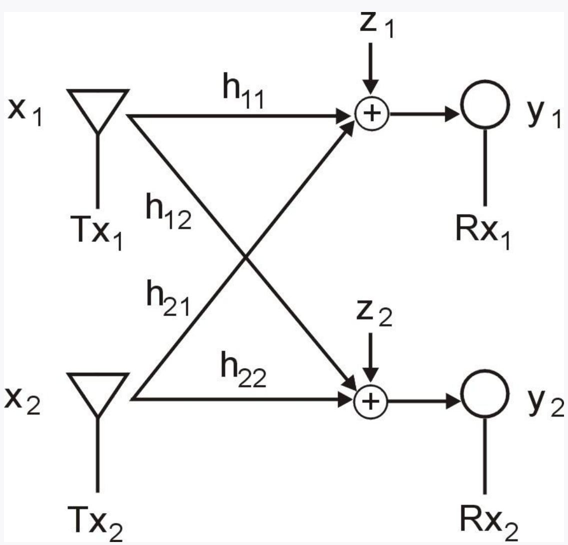
Deterministic





Interference

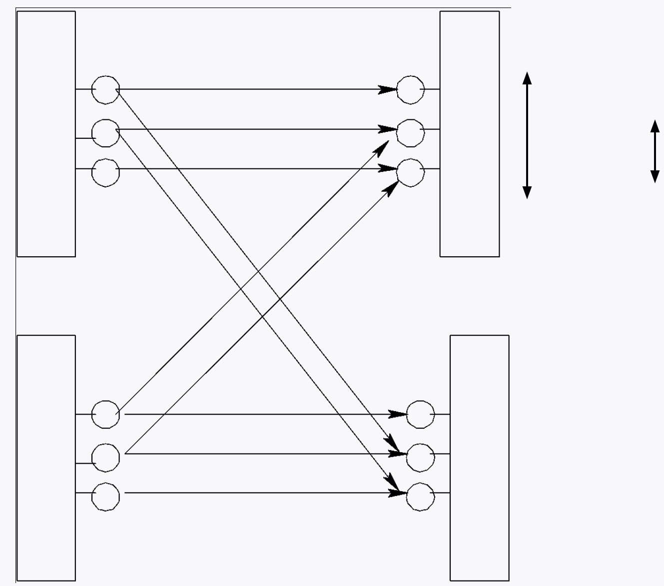
Gaussian



In symmetric case, channel described by two parameters:

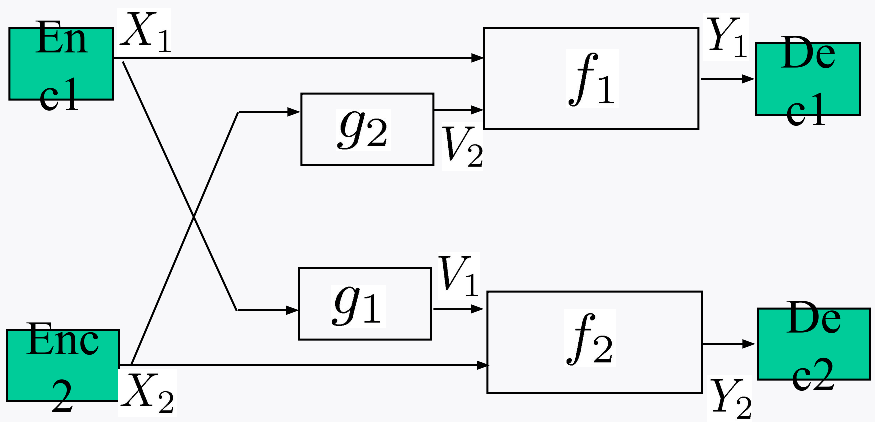
SNR, INR

Deterministic



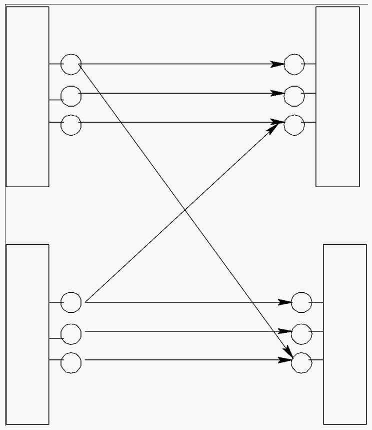
Capacity can be computed using a result by El Gamal and Costa 82.

Applying El Gamal and Costa



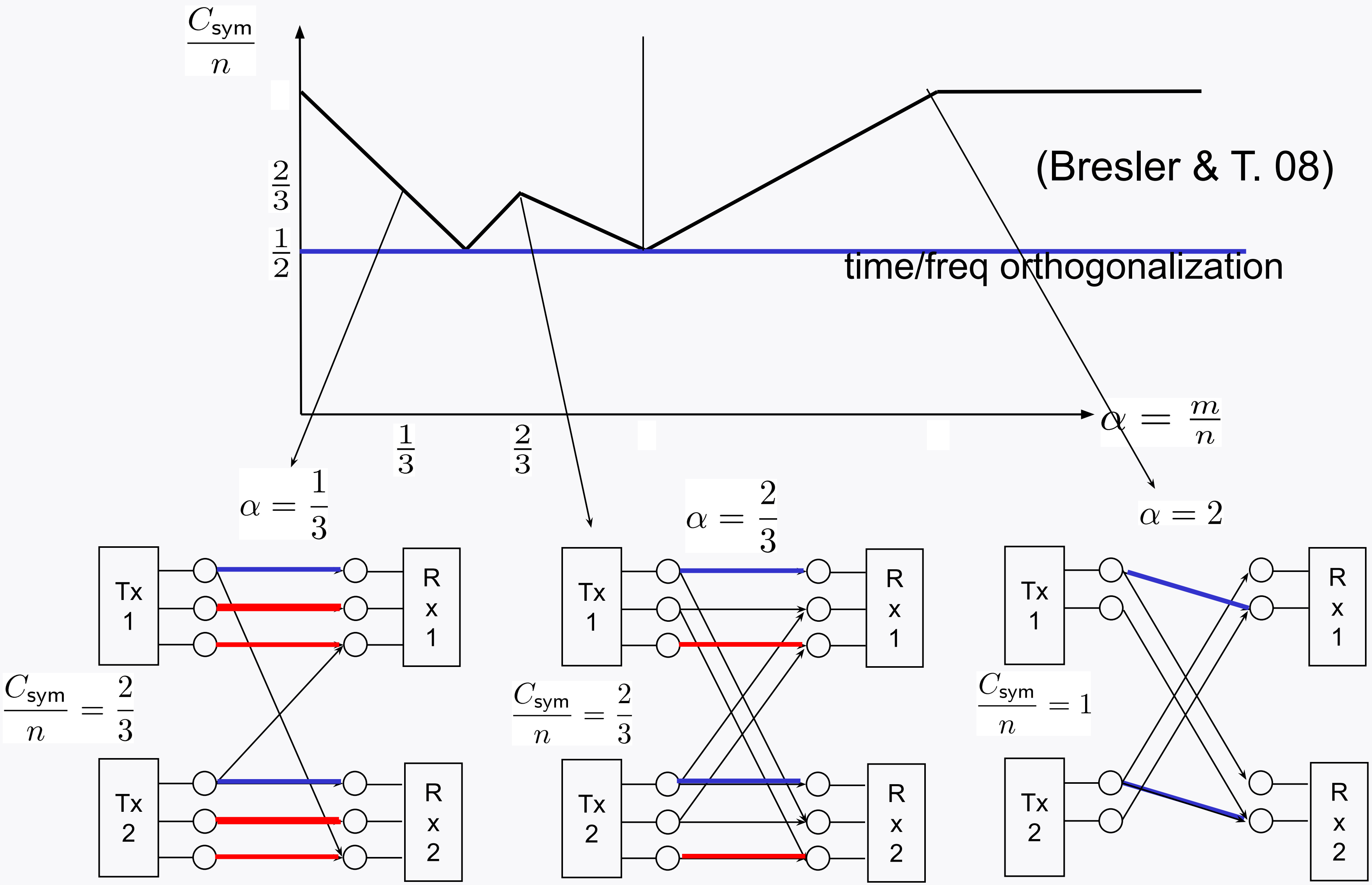
Han-Kobayashi with V_1, V_2 as common information is optimal.

Optimal inputs X_1^*, X_2^* uniform on the input alphabet.



Simultaneously maximizes all entropy terms.

Symmetric Capacity

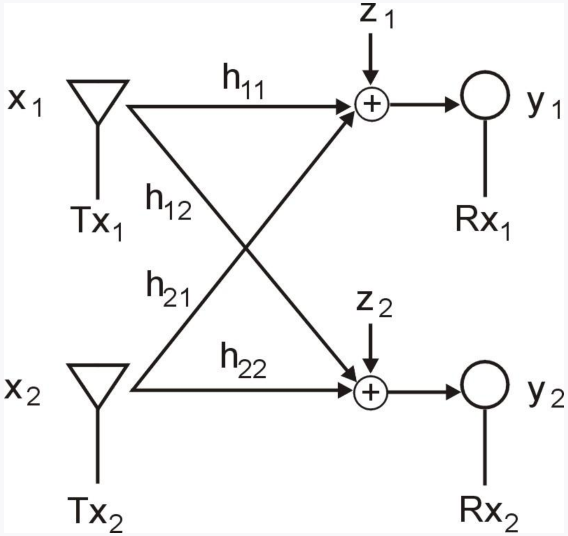


A Resource Sharing View

The two communication links share common resources via interference.

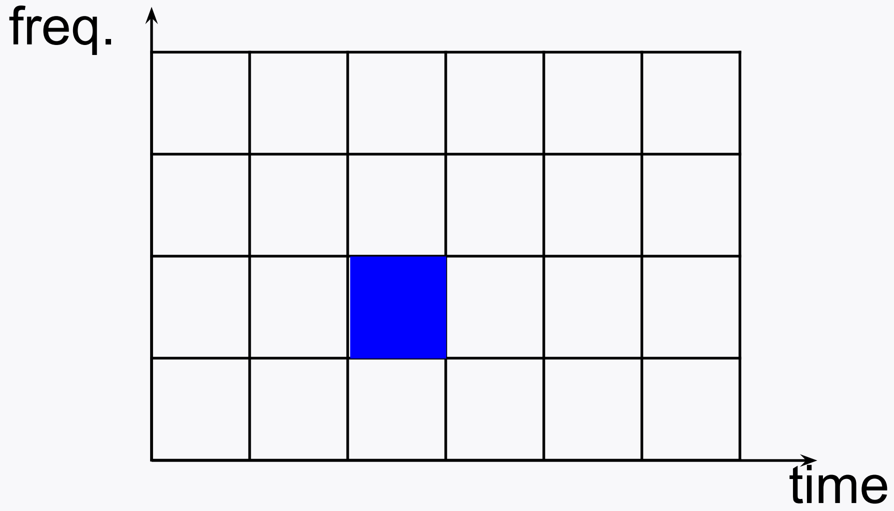
But what exactly is the resource being shared?

We can quantify this using the deterministic model.



Resource: Traditional View

time-frequency grid as a common ether.

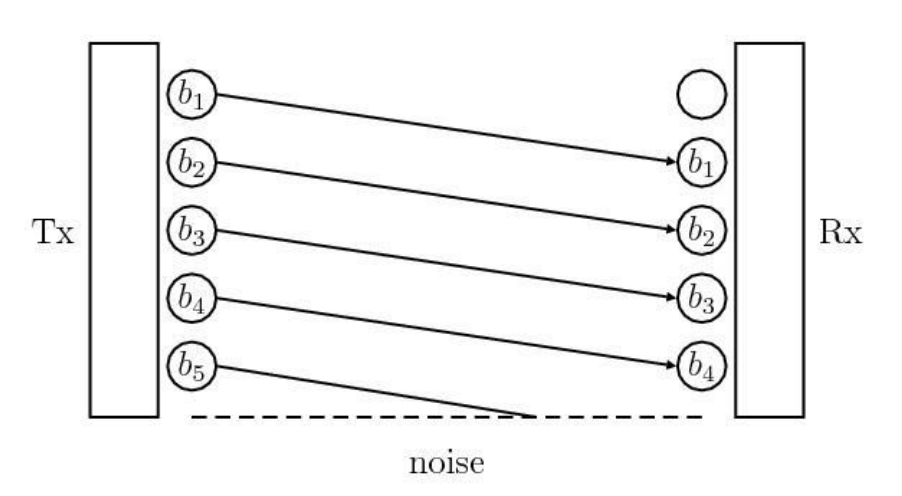


Each transmission costs one time-frequency slot.

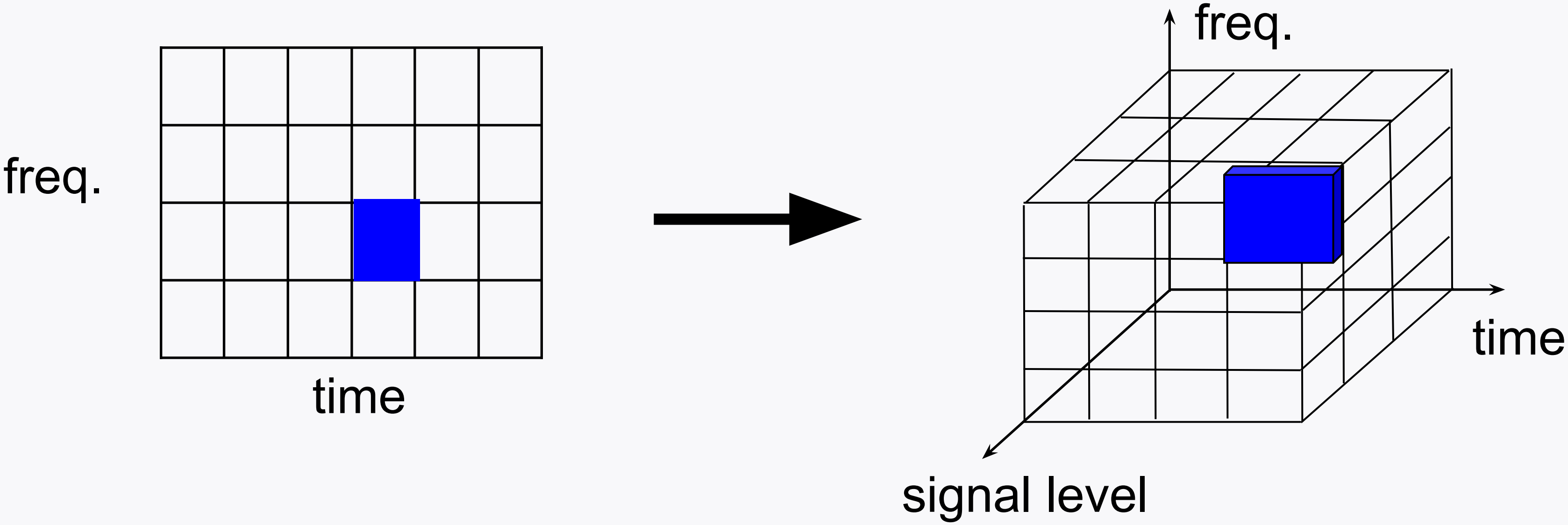
If a tree falls in a forest and no one is around to hear it, does it make a sound?

Resource is at the Receivers

- The action is at the receivers.
- No common ether: each Rx has its own resource.
- Signal strengths have to come into picture.
- Signal level provides a new dimension.



A New Dimension



Resource at a receiver:

# of resolvable bits per sample £ bandwidth £ time

W T

Resource and Cost

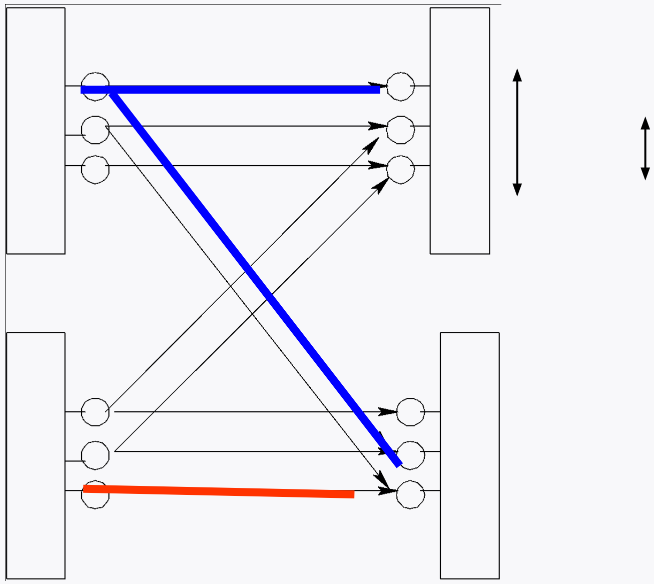
Resource available at each Rx

= max(m,n) signal levels ($)

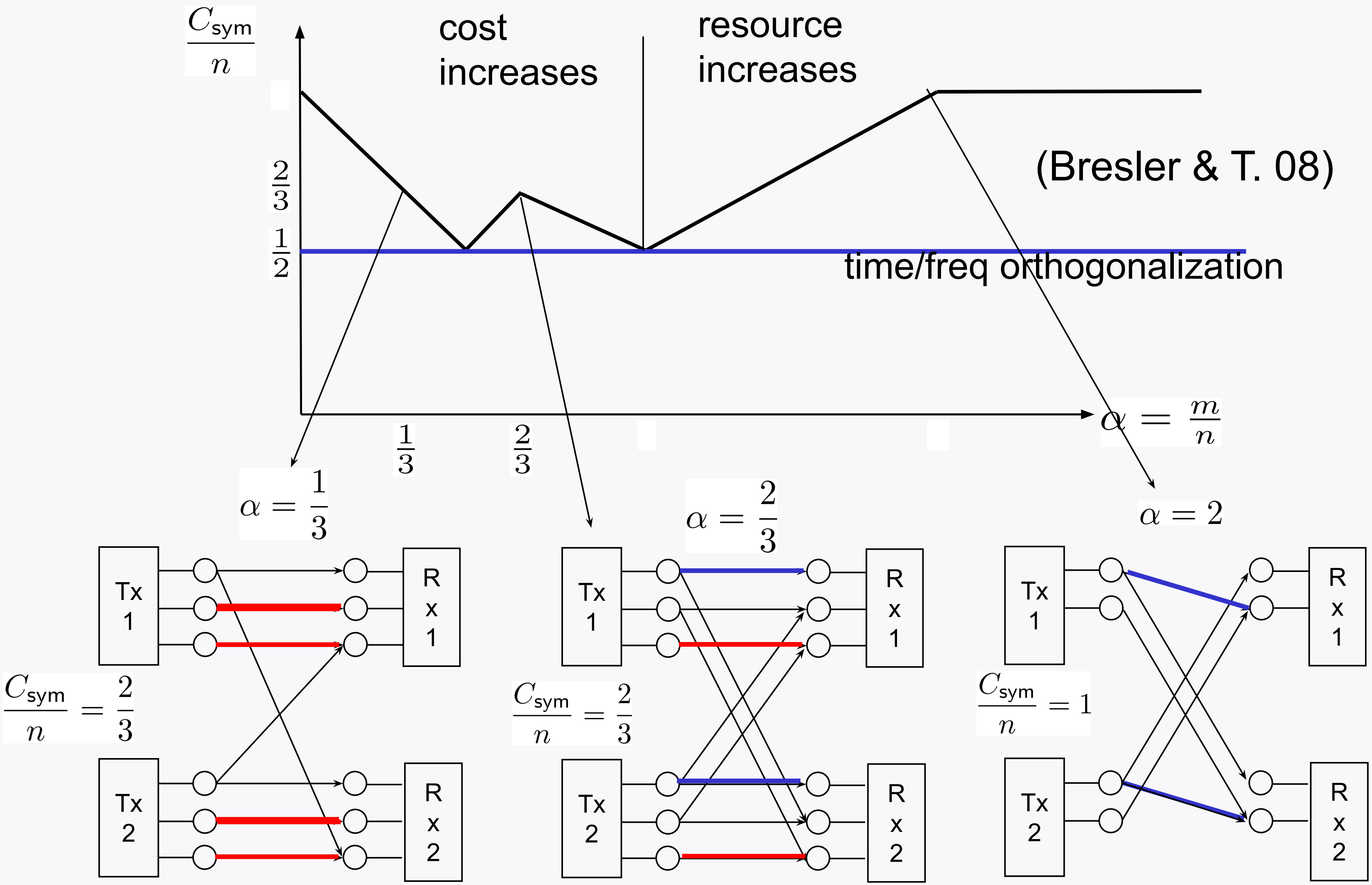
Cost to transmit 1 bit:

= $2 if visible to both Rx.

= $1 if visible to only own Rx.



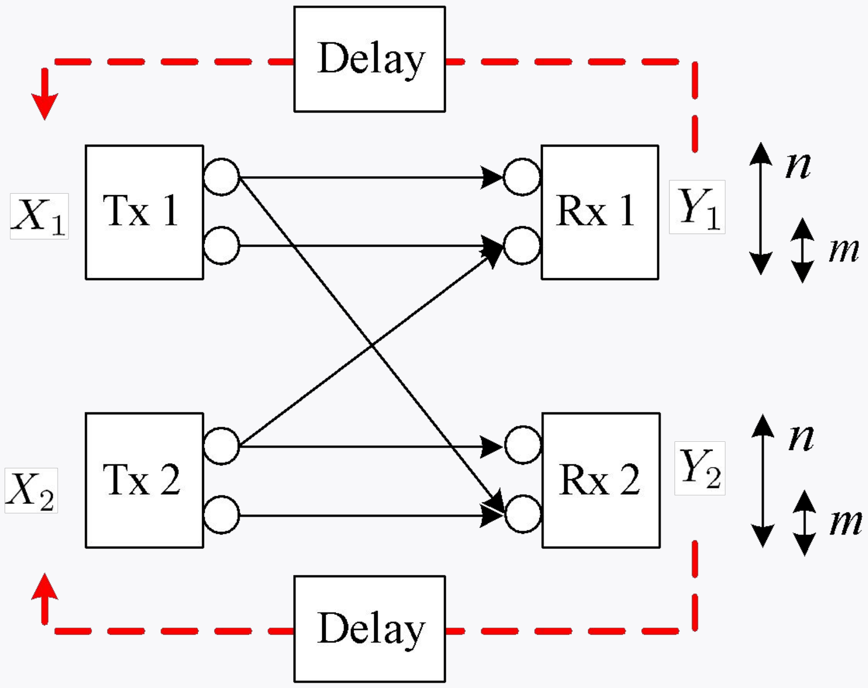
Symmetric Capacity



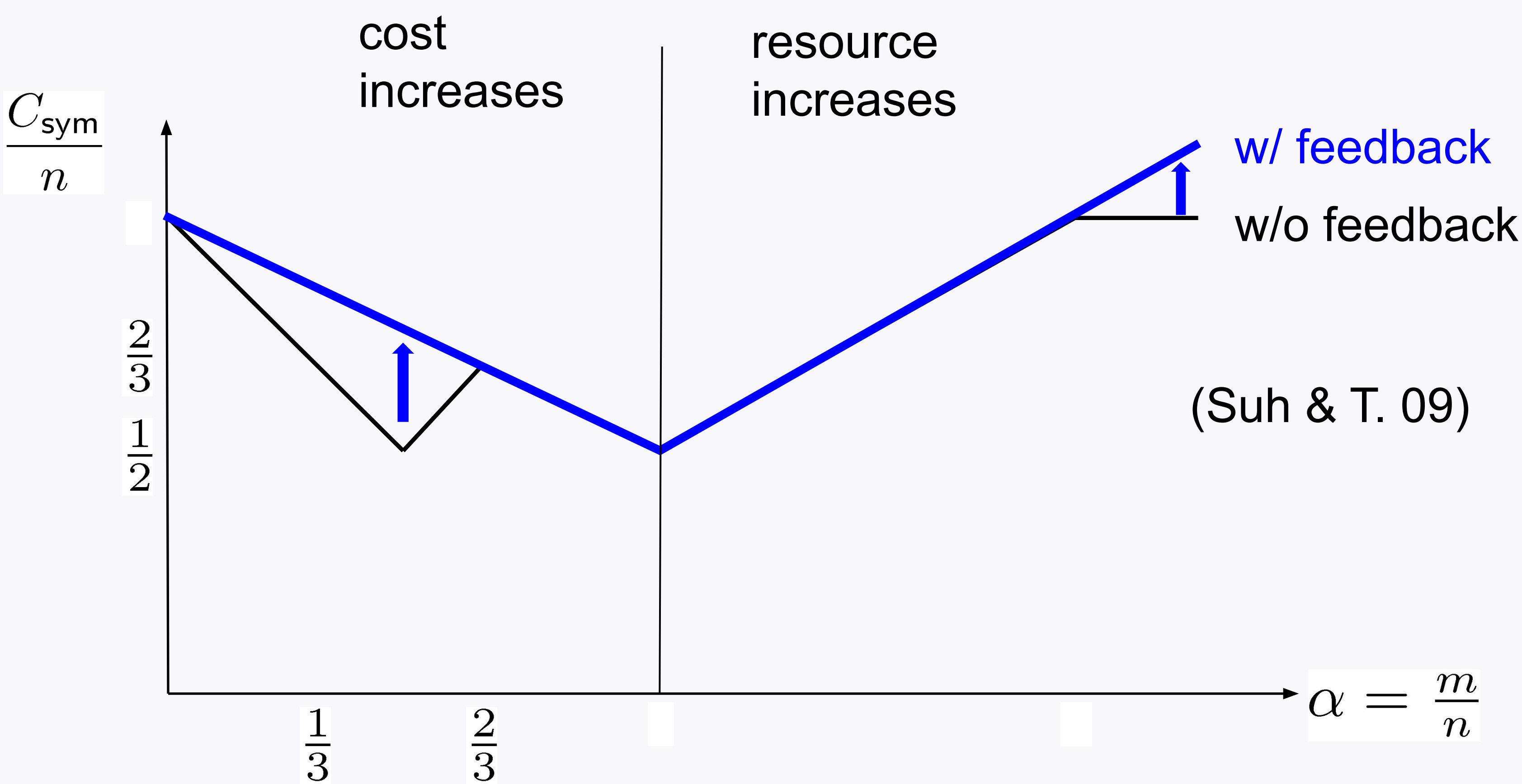
Follow-Up Questions

How does feedback and cooperation improve resource utilization?

Feedback



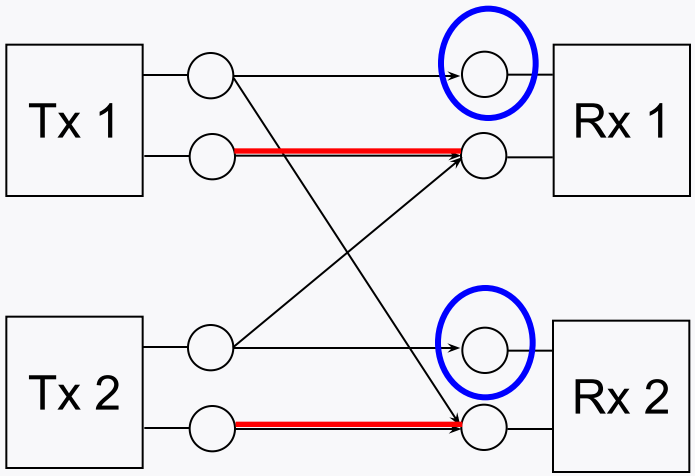
Can Feedback Help?

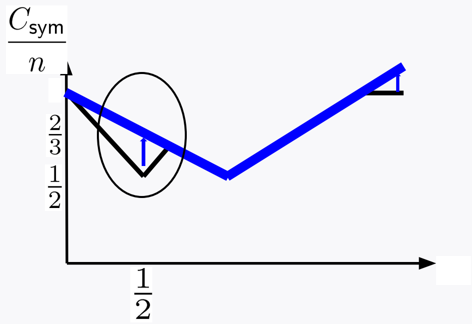


Feedback does not reduce cost, but it maximizes resource utilization.

Example: α = 0.5

w/o feedback



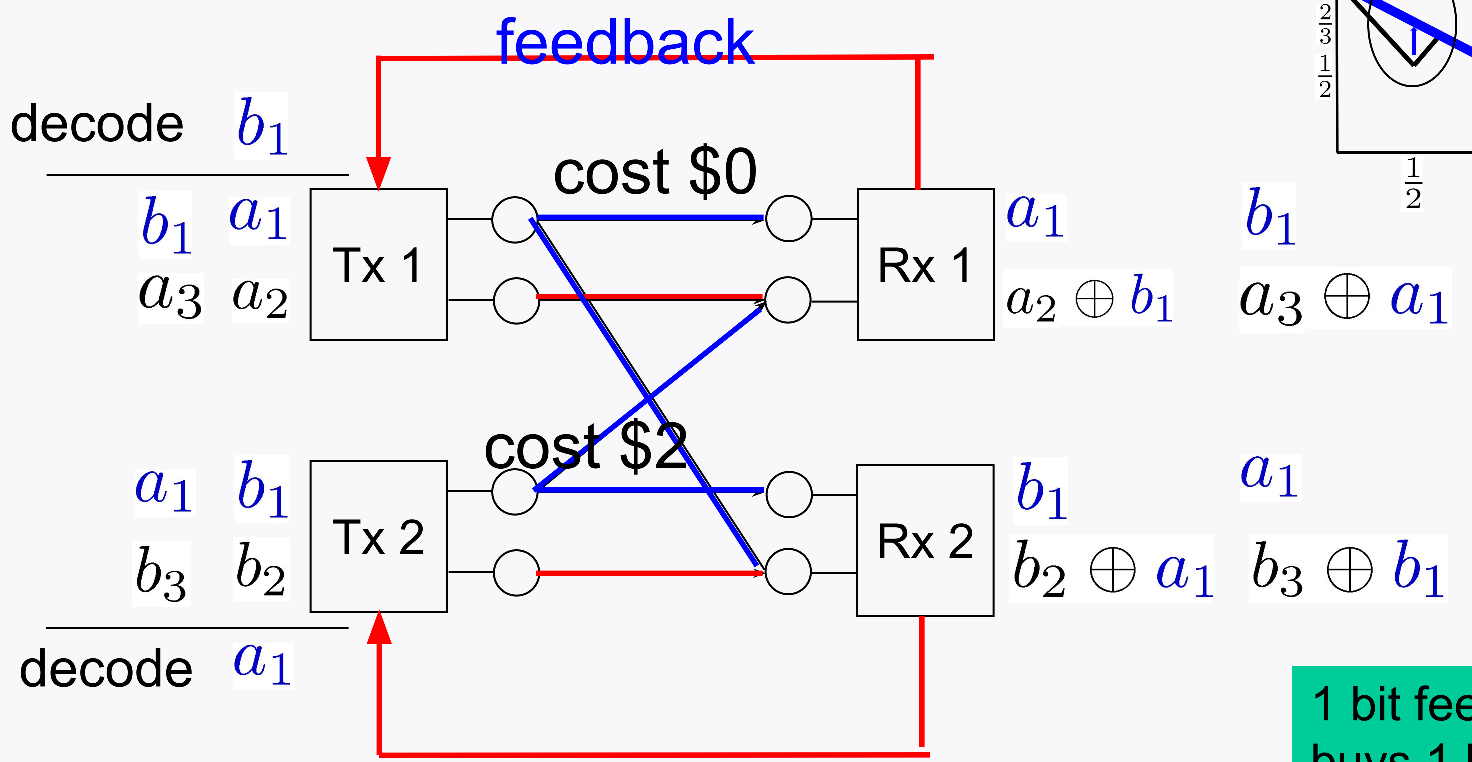


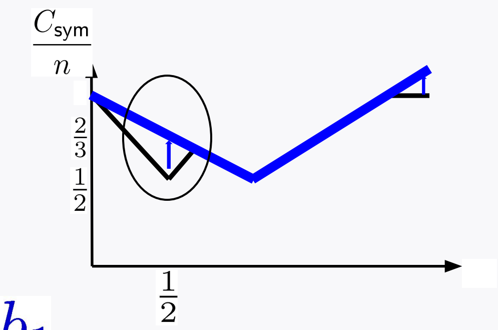
consumption: 2 levels

resource: 4 levels

Potential to squeeze 1 more bit in with feedback

Example: α = 0.5





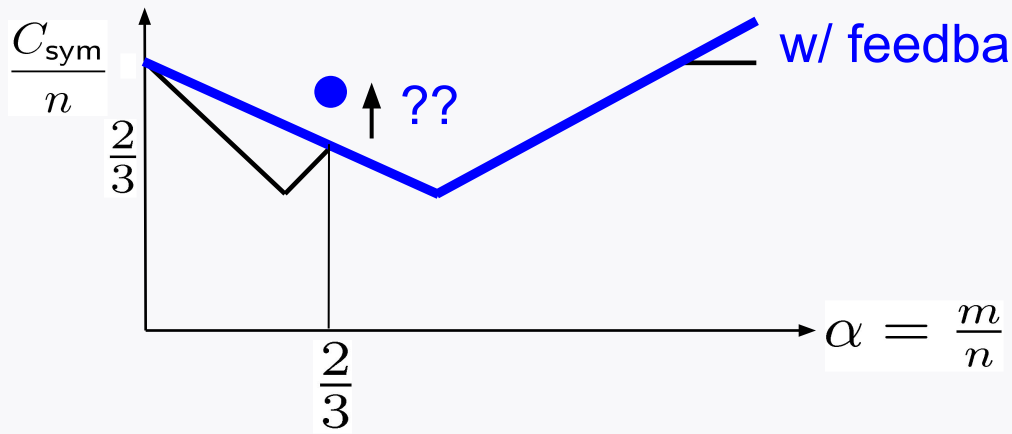
1 bit feedback buys 1 bit

Tx 1 sending b1 helps Rx 1 to recover a1 without causing interference to Rx 2.

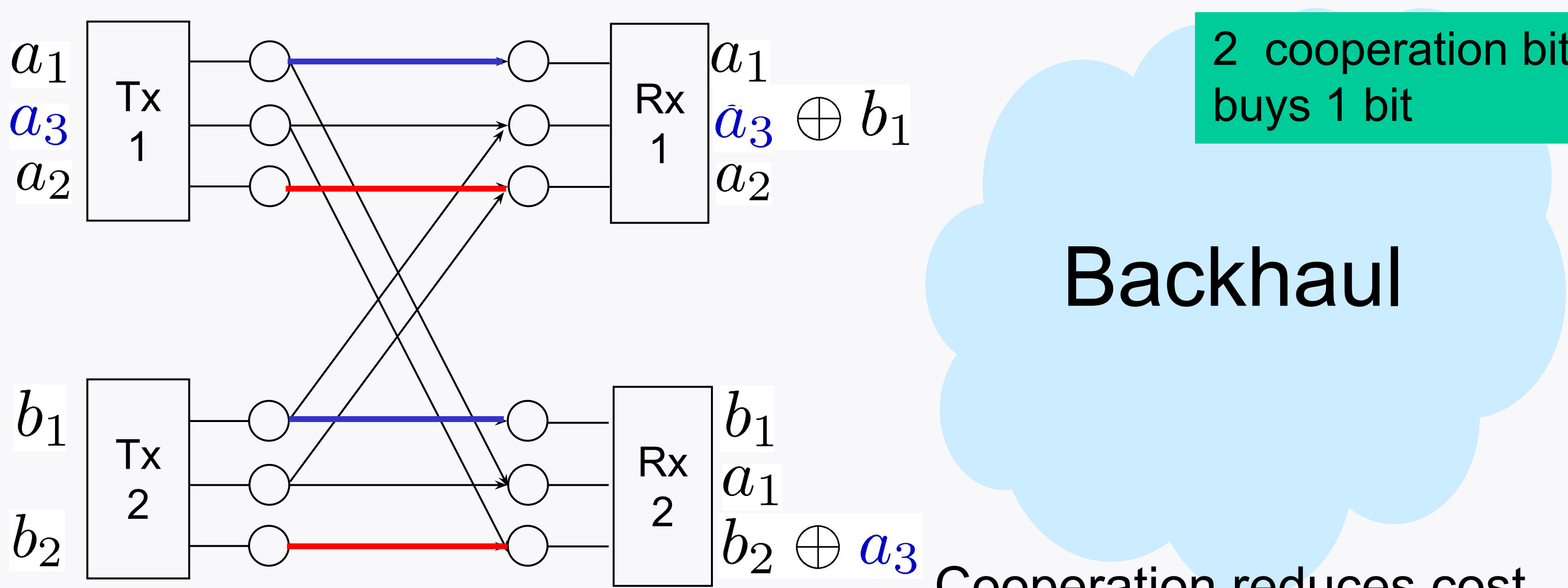
Gaussian Case

- There is a natural analog of this feedback scheme for the Gaussian case.
- Using this scheme, the feedback capacity of the 2-user IC can be achieved to within 1 bit/s/Hz.
- To find out, go to Changho Suh's talk on Thurs!

Can We Do Better than the V-curve?



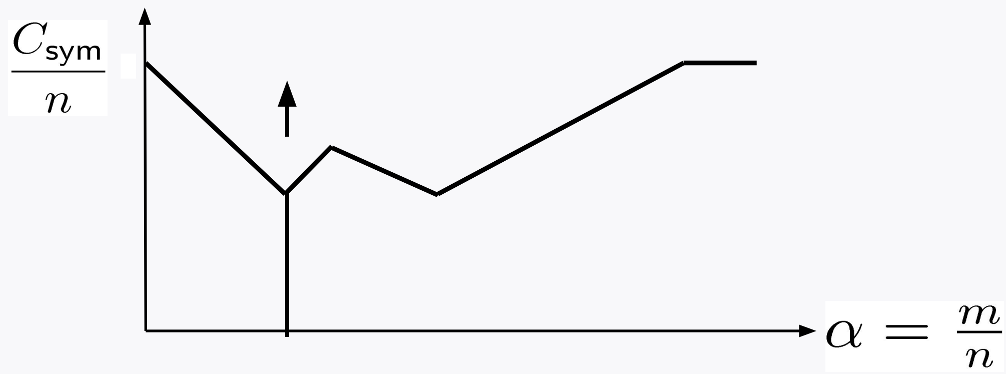
(Wang & T. 09)

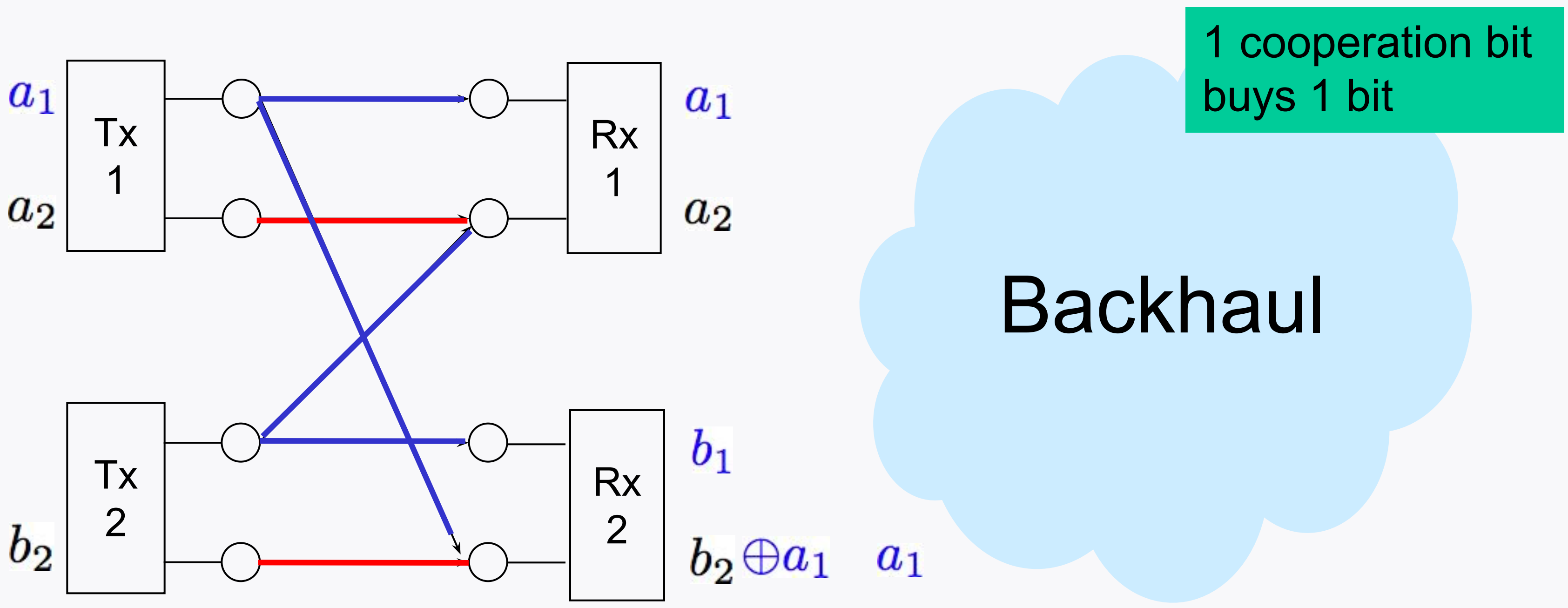


2 cooperation bits buys 1 bit

Cooperation reduces cost.

Cheaper Cooperation





Conferencing Capacity

- Devised a cooperation scheme for the Gaussian IC with conferencing decoders.
- Achieves capacity region to within 2 bits.
- Related work: cooperation via wireless links (Prabhakaran & Viswanath 08)

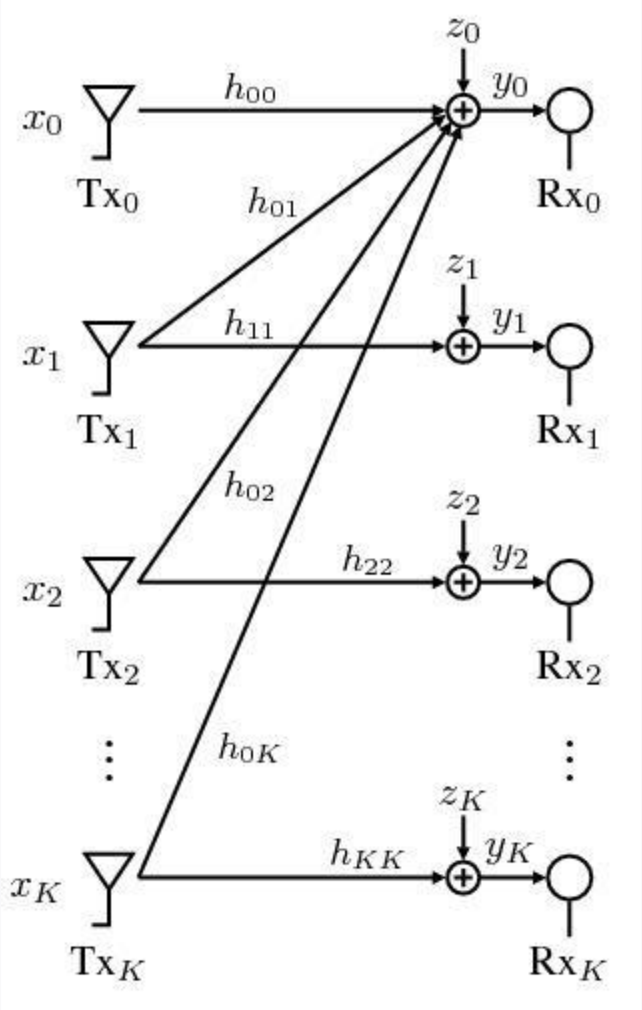
Part 3: Multiple Interferers and Interference Alignment

IC With More than 2 Users

- So far we have focused on the two-user interference channel.
- What happens where there are more than 2 users?
- Do the ideas generalize in a straightforward way?
- Not at all.
- We are far from a complete theory for K-user IC’s.
- We will go through a few examples to get a sense of what’s going on.

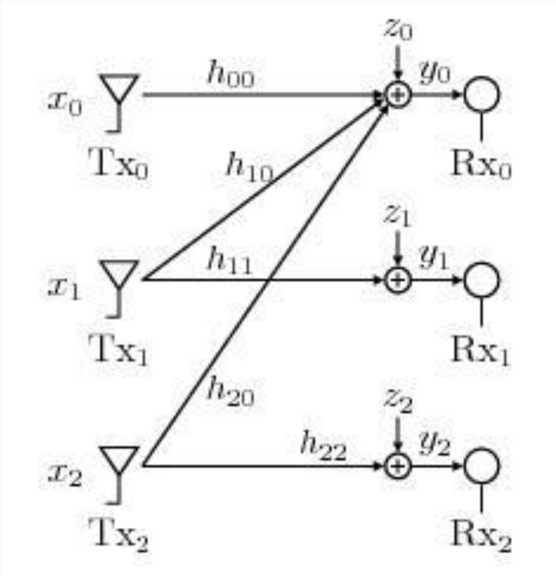
Many-to-One IC

- In the 2 user case a Han-Kobayashi achievable scheme with Gaussian inputs is 1-bit optimal.
- Is Han-Kobayashi scheme with Gaussian inputs optimal for more than 2 users?

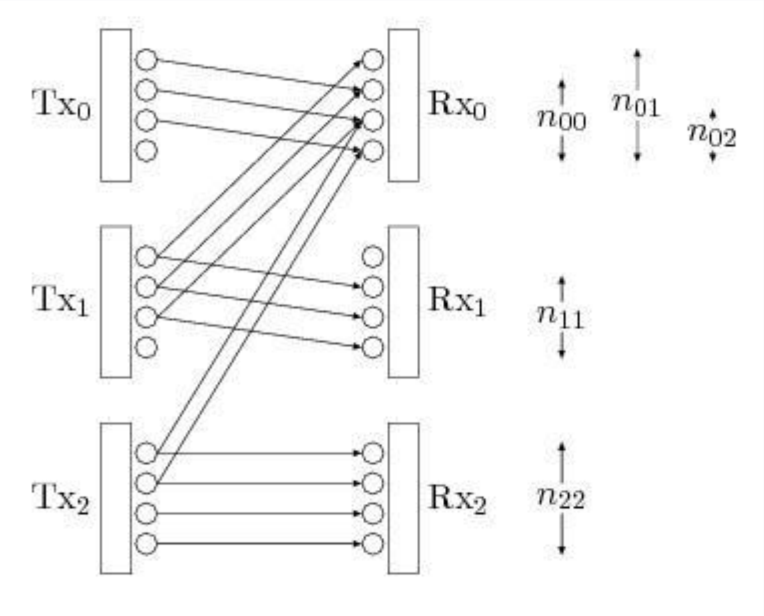


Deterministic Many to One IC

Gaussian



Deterministic



$$n_{ii} = \log_2 \mathtt{SNR}_i, \quad 0 \leq i \leq K$$

$$n_{0i} = \log_2 \mathtt{INR}_i, \quad 1 \leq i \leq K$$

Achievable Scheme

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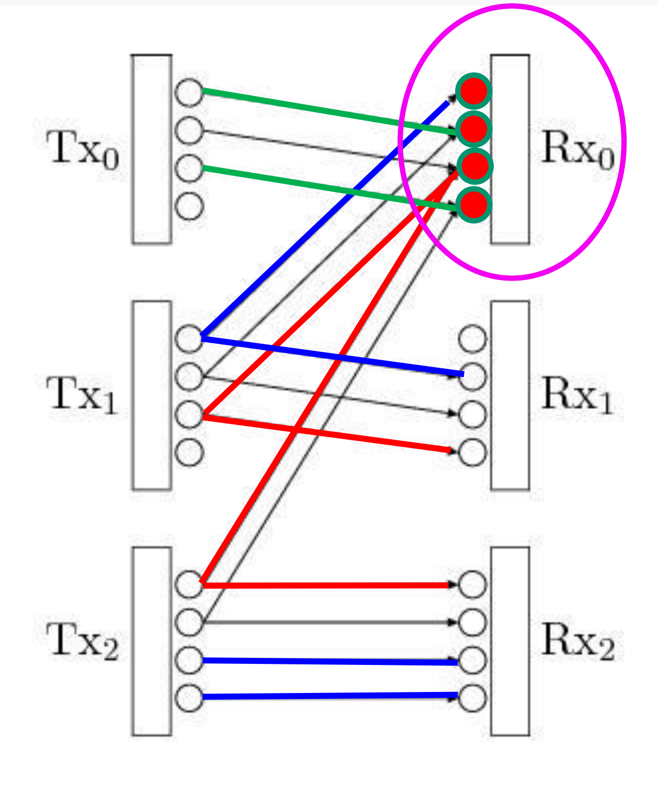
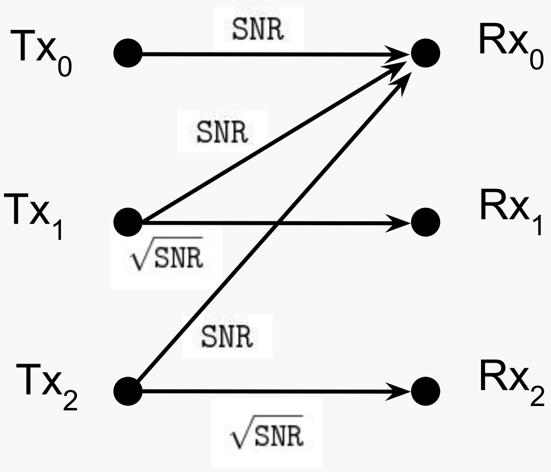
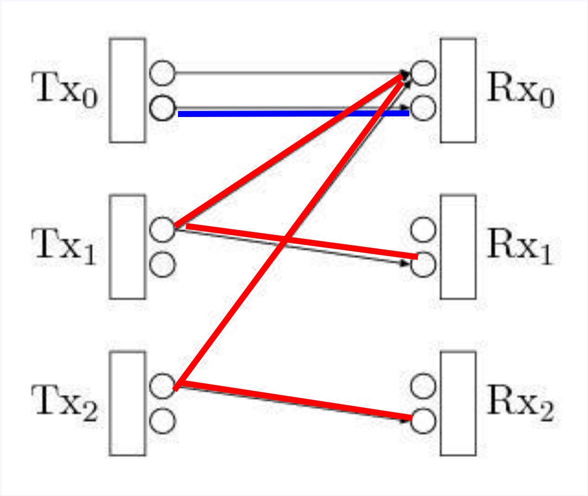
- Interference alignment: two (or more) users transmit on a level, cost to user 0 is same of that for a single interferer.
- Equivalently, cost of transmitting 1 bit for interferer is 1.5 levels.
- Turns out that scheme achieves capacity on the deterministic channel.



Example





- Interference from users 1 and 2 is aligned at the MSB at user 0's receiver in the deterministic channel.
- How can we mimic it for the Gaussian channel ?

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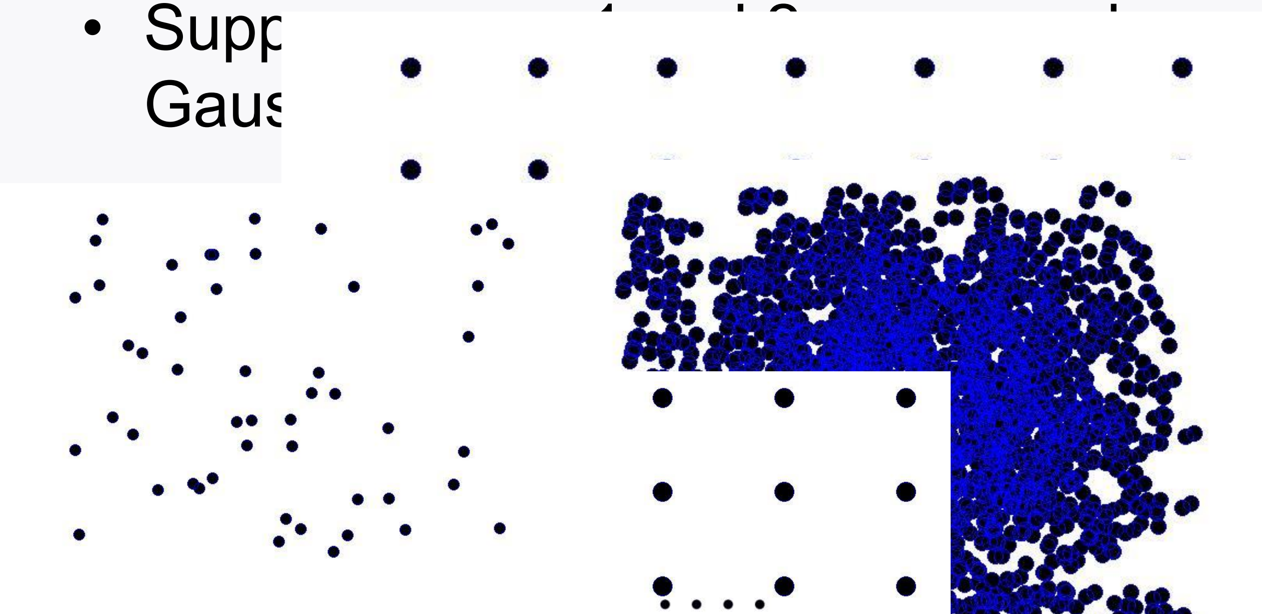
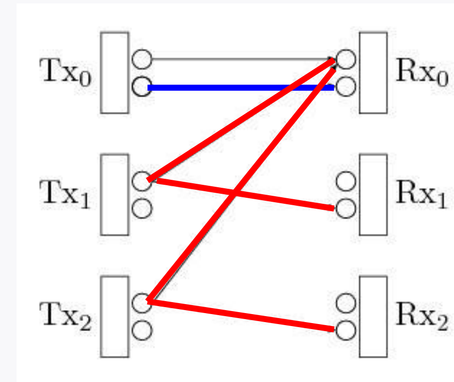
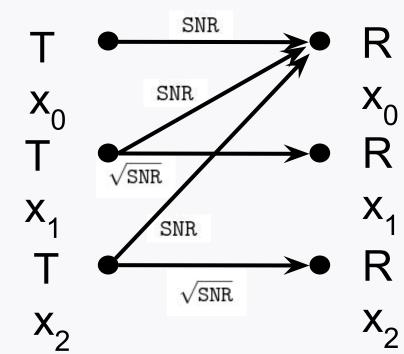


Random Code

Sum of Two Random Codebooks

Lattice Code for Users 1 and 2





Interference from users 1 and 2 fills the space: no room for user 0.

User 0 Code

Approximate Capacity

Theorem: (Approximate Capacity of K-user Many-to-One Gaussian IC).

Achievable scheme is within $\log_2 K$ bits of capacity, for any channel gains.

(Bresler, Parekh and T. 07)

What Have we Learnt

- In two-user case, we showed that an existing strategy can achieve within 1 bit to optimality.

- In many-to-one case, we showed that a new strategy can do much better.

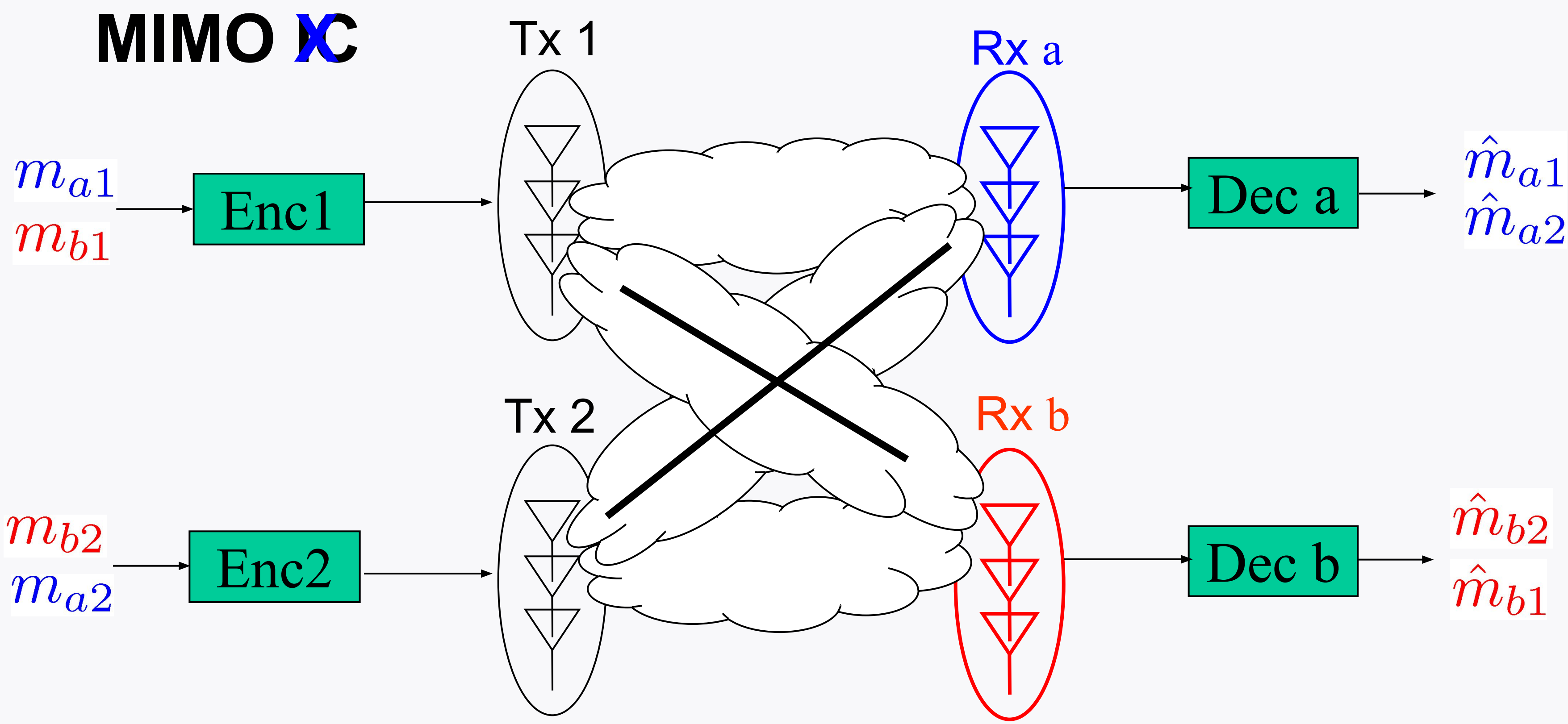
- Two elements:
  - Structured coding instead of random coding
  - Interference alignment

Interference Alignment: History

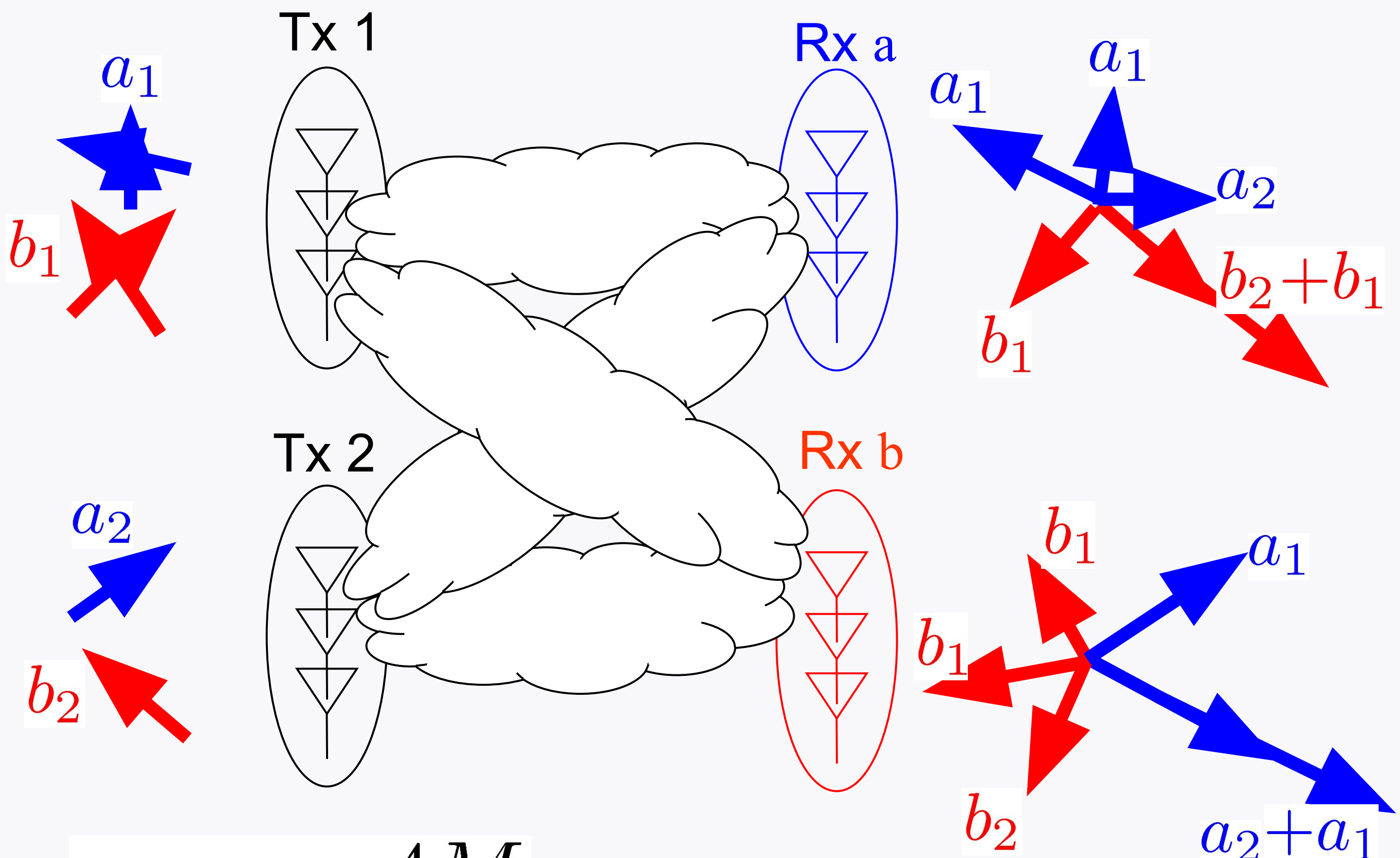
- First observed in the analysis of the X-Channel (Maddah-Ali et al 06)
- Concept crystallized by Jafar & Shamai 06
- Applied to the K-user parallel interference channel (Cadambe & Jafar 07)
- Applied to the many-to-one scalar IC (Bresler et al 07)

- Two types of interference alignment:
  - along time/frequency/space dimension
  - along signal scale

2-User MIMO X Channel



2-User MIMO X Channel



$$\text{d.o.f.} = \frac{4M}{3}$$

MIMO X-Channel vs Interference Channel

total dof of a 2-user MIMO with M antennas:

Interference Channel: M

(Jafar and Fakhereddin 06)

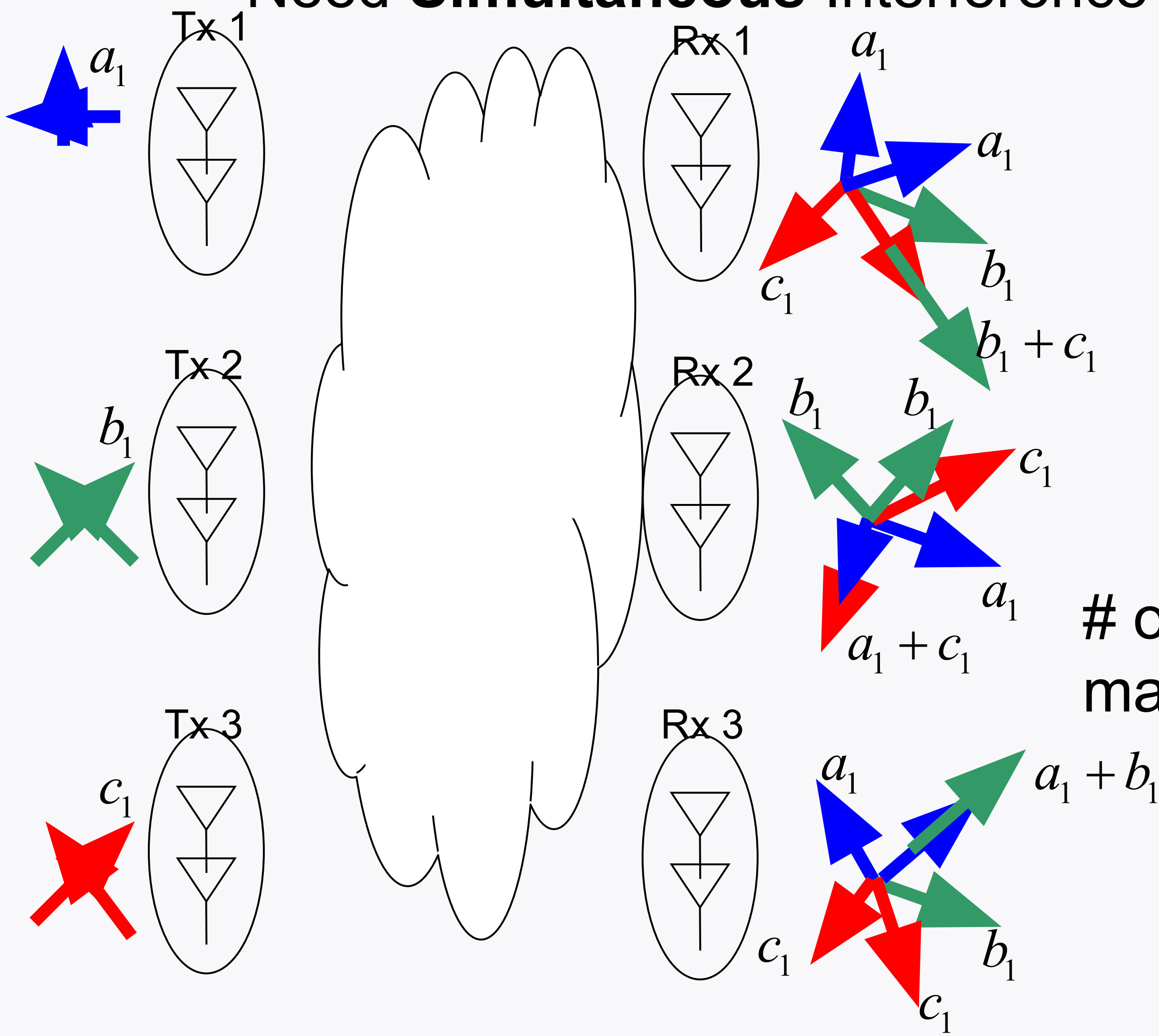
X- Channel: 4M/3

(Jafar and Shamai 06)

Interference alignment gain.

3-User MIMO IC

Need **Simultaneous** Interference Alignment



$$\mathbf{G}_{21}\mathbf{v}_2 = \mathbf{G}_{31}\mathbf{v}_3$$

$$\mathbf{G}_{12}\mathbf{v}_1 = \mathbf{G}_{32}\mathbf{v}_3$$

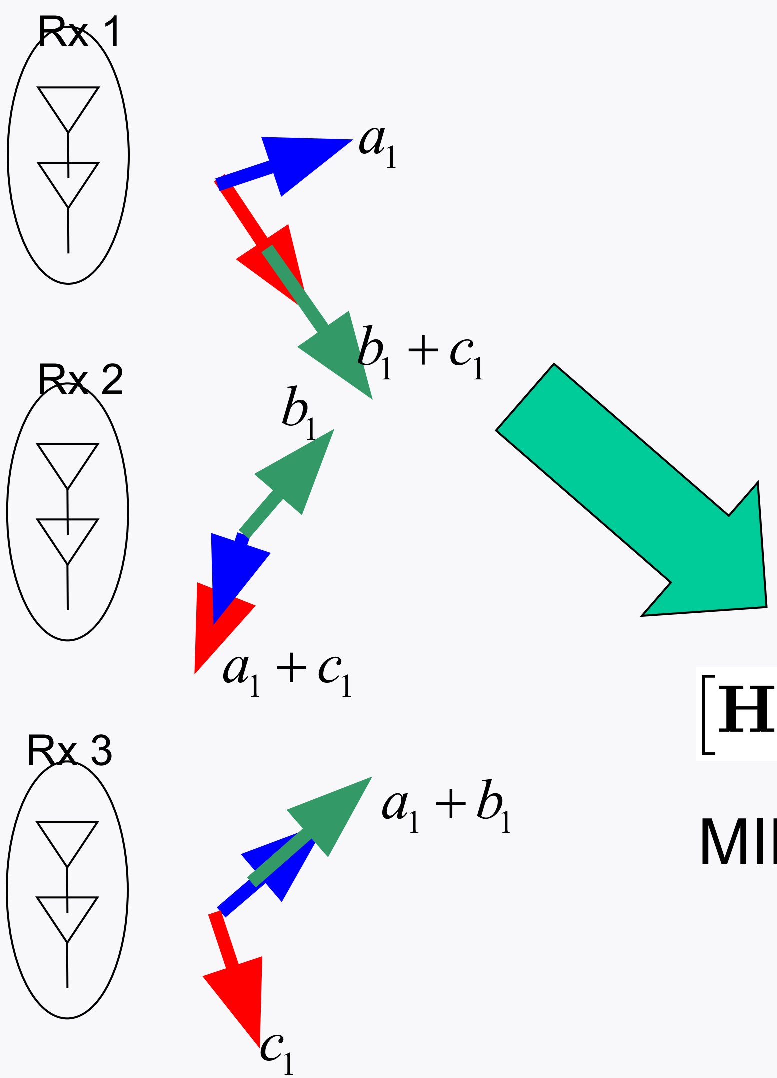
$$\mathbf{G}_{13}\mathbf{v}_1 = \mathbf{G}_{23}\mathbf{v}_2$$

3 conditions

3 vectors

# of conditions matches # of variables

3-User MIMO IC



$\mathbf{v}_1$:**eigenvector** of

$$\mathbf{G}_{12}^{-1}\mathbf{G}_{32}\mathbf{G}_{31}^{-1}\mathbf{G}_{21}\mathbf{G}_{23}^{-1}\mathbf{G}_{13}$$

$$\mathbf{v}_2 = \mathbf{G}_{23}^{-1}\mathbf{G}_{13}\mathbf{v}_1$$

$$\mathbf{v}_3 = \mathbf{G}_{31}^{-1}\mathbf{G}_{21}\mathbf{G}_{23}^{-1}\mathbf{G}_{13}\mathbf{v}_1$$

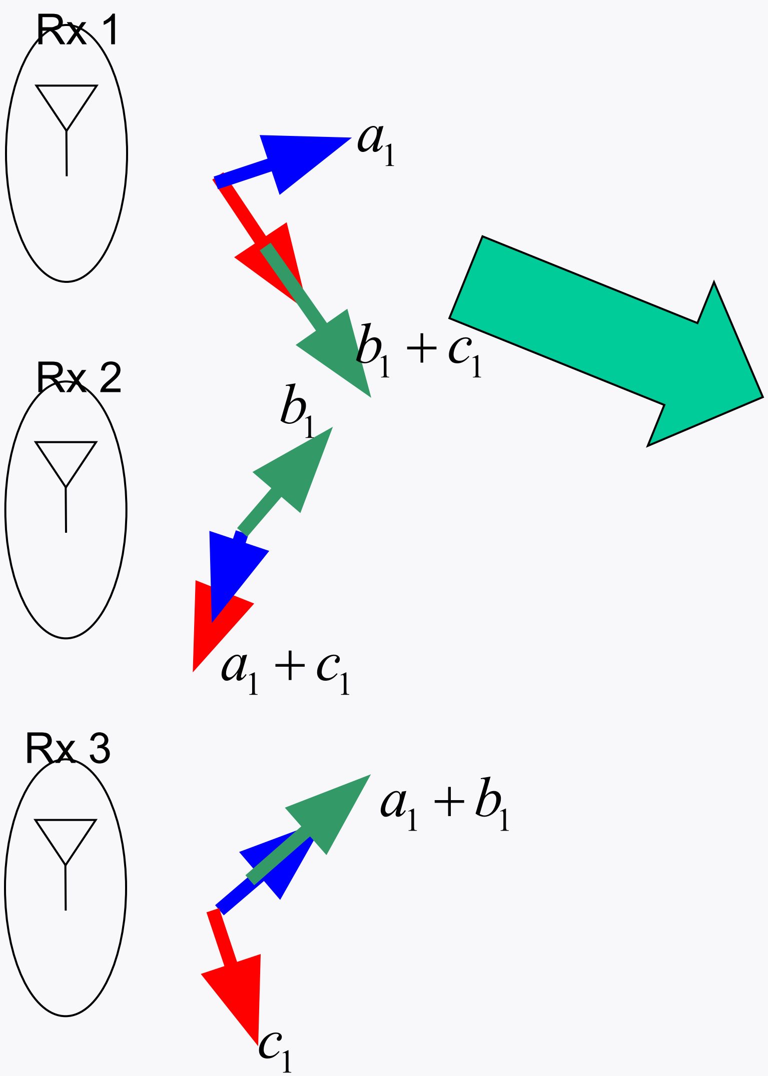
Check rank condition:

$$\left[\mathbf{H}_{11}\mathbf{v}_1 \quad \mathbf{G}_{21}\mathbf{G}_{23}^{-1}\mathbf{G}_{13}\mathbf{v}_1\right]$$

MIMO channel: rank=2 w.h.p.

3-User Parallel IC

Use 2 subcarriers



$\mathbf{v}_1$:**eigenvector** of

$$\mathbf{G}_{12}^{-1}\mathbf{G}_{32}\mathbf{G}_{31}^{-1}\mathbf{G}_{21}\mathbf{G}_{23}^{-1}\mathbf{G}_{13}$$

Check rank condition:

$$\left[\mathbf{H}_{11}\mathbf{v}_1 \quad \mathbf{G}_{21}\mathbf{G}_{23}^{-1}\mathbf{G}_{13}\mathbf{v}_1\right]$$

All matrices are diagonal.

$$\mathbf{v}_1 = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} \text{or} \begin{pmatrix} 0 \\ \alpha \end{pmatrix}$$

rank=1 

3-User IC: Summary

- With MIMO, can achieve optimal total dof of 3/2 per antenna.
- With finite number of parallel sub-channels, cannot.

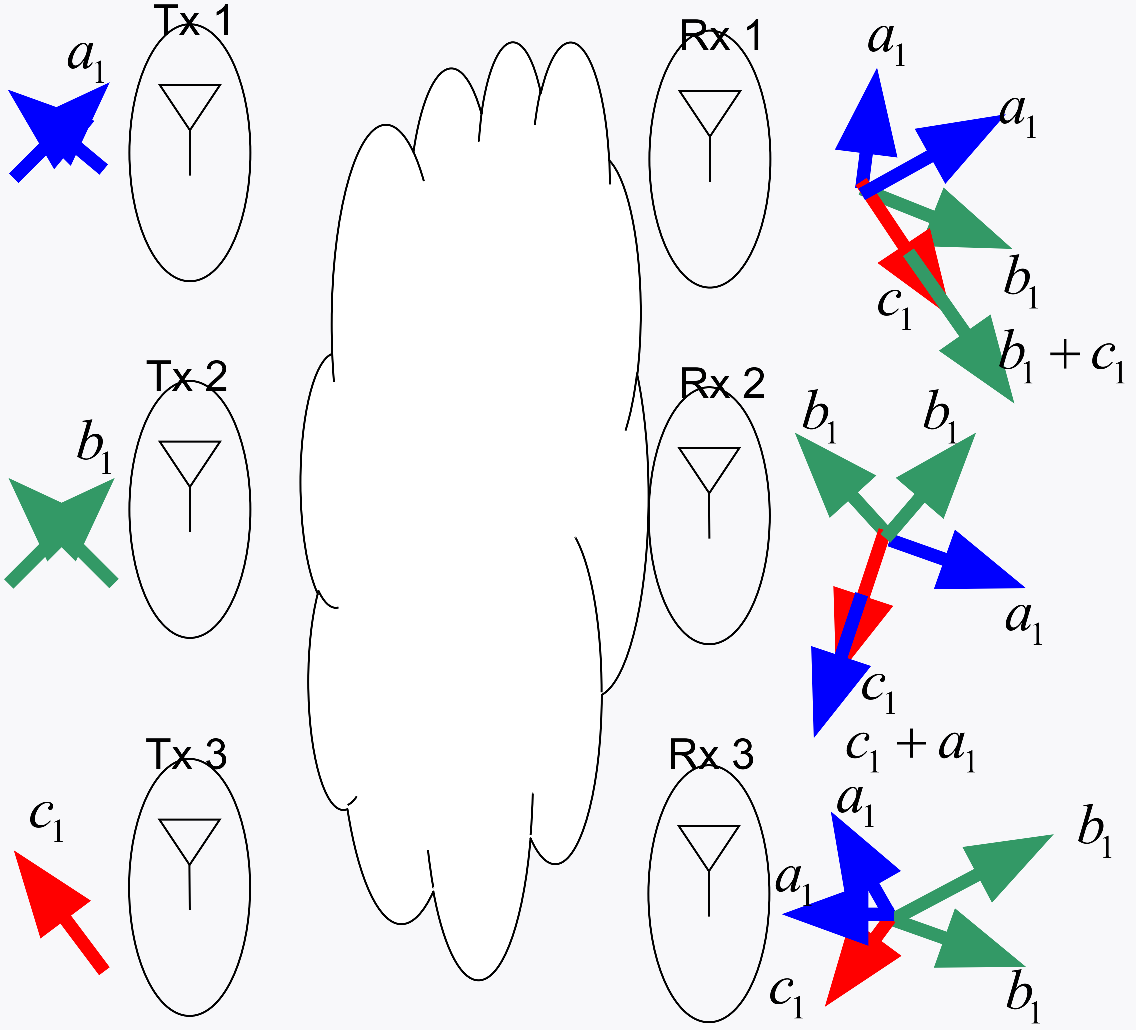
(Cadambe & Jafar 07)

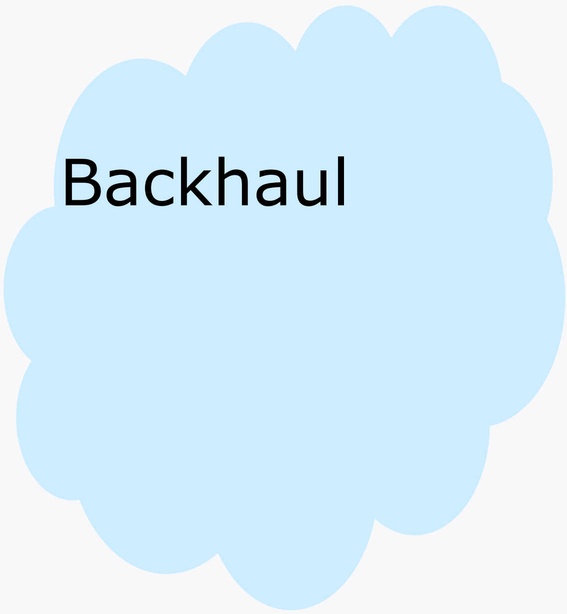
- As the number of parallel sub-channels grows, 3/2 can be achieved asymptotically.
- Key idea: partial subspace alignment
- In general, for K-user IC, K/2 can be achieved asymptotically.
- However, number of sub-channels scales like $(K^2)^{K2}$

Interference Alignment can still be useful

Use 2 subcarriers

Gallokota et al 09





Capacity

- For 2 user IC and many-to-one IC, we have constant gap capacity approximation.
- For 2-user X-channel and 3-user fully connected IC, we do not, even for single antenna.
- In fact, we don't even know the d.o.f.
- Interference alignment on signal scale is useful for very specific channel parameter values (Cadambe, Jafar & Shamai 08, Huang, Cadambe & Jafar 09, Etkin & Ordentlich 09)
- But we don't know if it's useful for many parameter values.

Conclusions

- A good understanding of the 2-user IC, even with feedback and cooperation.
- Deterministic modeling is a useful technique.
- Interference alignment has been shown to be a useful technique when there are multiple interfererers.
- But we don’t have a good understanding on the capacity when there are multiple interferers.